Econ 73-250A-F
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Suggested Answers to Problem Set #2

Please refer to the end of this document for all graphs.

Exercise #1. (a)

$$MU_1(x_1,x_2) = \frac{\partial U(x_1,x_2)}{\partial x_1} = (x_2+1)^2.$$

(b)

$$MU_2(x_1,x_2) = \frac{\partial U(x_1,x_2)}{\partial x_2} = 2x_1(x_2+1).$$

(c)

$$MRS(x_1,x_2) = -\frac{\frac{\partial U(x_1,x_2)}{\partial x_1}}{\frac{\partial U(x_1,x_2)}{\partial x_2}} = -\frac{(x_2+1)^2}{2x_1(x_2+1)} = -\frac{(x_2+1)}{2x_1}.$$

Anna's MRS at point (3,5) is $-(5+1)/2(3) = -1$.

(d) Anna's utility at the point (3,5) is $u(3,5) = 108$. If Anna exchanges 1 unit of good 2 for 1 unit of good 1, she obtains the bundle (4,4). Anna's utility from consuming this bundle is $u(4,4) = 100 < 108$. Anna is therefore not willing to exchange 1 unit of good 2 for 1 unit of good 1 (assuming that she is currently consuming the bundle (3,5)). This finding appears to be at odds with the fact that Anna's MRS at the point (3,5) is 1, meaning that Anna is just willing to exchange 1 unit of good 2 for 1 unit of good 1. But remember that the MRS measures Anna's willingness to exchange good 2 for good 1 only for small trades. Put differently, the MRS is a local measure of the slope of an individual's indifference curve near a given consumption bundle.

(e) Suppose that Anna's consumption bundle is $(27,x_2)$. Then, her utility from consuming this bundle is

$$u(27,x_2) = 27(x_2+1)^2.$$

Set this quantity equal to 108 (which is the utility that Anna obtains from the bundle (3,5)) and solve for x_2:

$$27(x_2+1)^2 = 108 \Rightarrow (x_2+1)^2 = 4 \Rightarrow x_2+1 = 2 \Rightarrow x_2 = 1.$$

Thus, the bundles (3,5) and (27,1) are on the same indifference curve. If Anna has convex preferences, then (the absolute value of) her MRS at (27,1) should be lower than (the absolute value of) her MRS at (3,5), since the bundle (27,1) is farther down the indifference curve than the bundle (3,5). This is indeed the case: using the result from part (c), (the absolute value of) her MRS at (27,1) is 2/54, or 1/27, while (the absolute value of) her MRS at (3,5) is 1.

Exercise #2. (a) The government budget line has the following equation

$$10 = x_d + 50x_h.$$

In this example we cannot strictly talk about prices for the two goods x_d and x_h. However, we can talk about the opportunity cost of reducing the debt. In fact, for each extra million of dollars that is spent to pay back the debt, the government's health plan will be able to cover a smaller fraction x_h of the cost of a visit to a doctor. How much smaller? To see this re-write the budget line in the following way

$$x_h = \frac{1}{5} - \frac{1}{50}x_d$$

For each extra million of dollars spent in x_d, the government has to reduce x_h by $1/50 = 0.02$. This number is the "relative price" or opportunity cost of x_d in terms of x_h.

(b) The prime minister has Cobb-Douglas preferences over bundles (x_h, x_d). The budget line is

$$10 = x_d + 50x_h$$

where 10 is the prime minister's "income" (call it m), 1 the nominal (dollar) price (call it p_d) of the good "millions of dollars spent in paying back the debt" and 50 is the nominal price (call it p_h) of the good "percentage of the cost of a visit to a doctor paid by the government".

As we have seen in class the optimal choice for a consumer with Cobb-Douglas preferences is, in general,

$$\begin{aligned} x_h^* &= c\frac{m}{p_h}, \\ x_d^* &= (1-c)\frac{m}{p_d} \end{aligned}$$

where c is the exponent of x_h in the Cobb-Douglas utility function, which in our case is $1/2$. Making the relevant substitutions, we obtain

$$\begin{aligned} x_h^* &= c\frac{m}{p_h} = \frac{1}{2}\frac{10}{50} = \frac{1}{10} = 0.10, \\ x_d^* &= (1-c)\frac{m}{p_d} = \frac{1}{2}\frac{10}{1} = 5. \end{aligned}$$

Thus the prime minister will choose to spend \$5 millions to pay back the debt and let the government's health plan pay 10% of the cost of a visit to a doctor.

(c) As we have seen in (b), the government pays 10% of the cost of a visit to a doctor. The full cost of a doctor's visit is \$100, so the government pays \$10 for each visit. The total number of visits are 500,000, since half of the population is expected to make an appointment with a doctor. Thus the total cost for the government is

$$\$10 \times 500{,}000 = \$5{,}000{,}000.$$

This represents a share of $1/2$ of the government budget (that equals \$10 millions). Alternatively we could have noticed that with Cobb-Douglas preferences, the exponent c of x_h in the utility function measures the share of income that is spent in x_h. This share is 1/2.

(d) The answer would not change because the utility function $v(x_d, x_h)$ represents the same preferences as the utility function $u(x_d, x_h)$. In fact, the utility function $v(x_d, x_h)$ is the result of the following positive monotonic transformation of $u(x_d, x_h)$

$$v(x_d, x_h) = \log\left(\frac{u(x_d, x_h)}{1000}\right).$$

Since the underlying preferences are the same, the prime minister's choices do not change. He will spend 1/2 of the government budget in health care.

Exercise #3. (a) Nancy's time constraint is

$$400 = m_1 + m_2.$$

The exercise asks for the budget line in terms of x_1 and x_2. Substituting into this equation the expressions for m_1 and m_2 we get

$$400 = 5x_1 + 10x_2.$$

We can simplify this dividing by 5 to get the "budget line"

$$80 = x_1 + 2x_2.$$

Since Nancy's overall grade depends only on the maximum of the two grades, her utility function can be represented in the following way

$$u(x_1, x_2) = \max(x_1, x_2).$$

Of course any positive monotonic transformation of $u(x_1, x_2)$ also represents the same preferences.

To draw an indifference curve set $\max(x_1, x_2) = \overline{u}$, where $\overline{u}$ is a number. For example, set $\overline{u} = 1$, and ask yourself what are the bundles (x_1, x_2) that satisfy $\max(x_1, x_2) = 1$. To plot this, first fix $x_1 = 1$, and notice that the equation for the indifference curve is satisfied as long as $x_2 \leq 1$. Then, fix $x_2 = 1$ and notice that the same equation is satisfied for $x_1 \leq 1$. We now have one indifference curve. Repeat the same operation for other values of $\overline{u}$ to get other indifference curves, plotted in Figure 1.

(b) Re-write Nancy's budget line as follows:

$$x_2 = 40 - \frac{x_1}{2}.$$

From this equation you can notice that for each extra score in the first examination, Nancy has to give up half score in the second examination. Thus, it is clearly optimal for her to dedicate all her time to the first examination, given that she only cares about the maximum score. Thus, her choice will be $X^* = (80, 0)$, obtained by setting $x_2 = 0$ in the budget line and getting $x_1 = 80$. Her choice is represented in Figure 1. Notice that in this exercise Nancy has concave preferences and, as we have seen in class, this leads her to consume only one of the goods.

(c) Her overall score is then

$$\max(80, 0) = 80.$$

Figure 1