Econ 73-250A-F
Spring 2001
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Solution to Problem Set #1

Please, find all the graphs at the end of this document.

Exercise #1.

(a) What is the relative price of hamburgers in terms of clothes in Concord in 1998?
The relative price of hamburgers in terms of clothes in Concord in 1998 is

$$\frac{\frac{\$0.5}{1 \text{ hamburger}}}{\frac{\$30}{1 \text{ cloth}}} \approx 0.016 \text{ clothes per hamburger.}$$

(b) Write down the equation that represents the Adams' budget constraint in 1998.
The Adams' budget constraint is

$$\underbrace{60,000\,(1-0.5)}_{\text{Adams' net income}} \geq 0.5x_h + 30x_c.$$

(c) The budget line has equation

$$x_h = 60,000 - 60x_c.$$

See Figure 1.

(d) A president is elected at the end of the year 1998 that reduces the federal income tax rate for the following years from 50% to 10%. Write down the equation that represents the Adams' new budget constraint in 1999:

$$\underbrace{60,000\,(1-0.1)}_{\text{Adams' net income}} \geq 0.5x_h + 30x_c.$$

The equation that represent the budget line reads

$$x_h = 108,000 - 60x_c.$$

This line is parallel to the one in point c. See Figure 2 for the rest.

(e) What is the relative price of hamburgers in terms of clothes in Pittsburgh in 2000?
The relative price of hamburgers in terms of clothes is

$$\frac{\frac{\$0.5}{1 \text{ hamburger}}}{\frac{\$30(1+0.07)}{1 \text{ cloth}}} \approx 0.015 \text{ clothes per hamburger.}$$

Are hamburgers more expensive in terms of clothes in Concord or in Pittsburgh? In Concord.

(f) Write down the equation that represents the Adams' budget constraint in 2000.

$$\underbrace{60,000\,(1-0.1)}_{\text{Adams' net income}} \geq 0.5x_h + 30\,(1+0.07)\,x_c.$$

(g) The budget line has equation

$$x_h = 108,000 - 64.2x_c.$$

See Figure 3 for the graph. How many dollars would the Adams pay in sales taxes in the year 2000, if they spent 30,000$ in hamburgers and the rest of their income (net of federal taxes) in clothes?

Their net income is $54,000. After spending $30,000 in hamburgers, they are left with $24,000. The latter amount is used to buy clothes (and to pay sales taxes). Thus, from their budget constraint:

$$\$24,000 = \$30\,(1+0.07)\,x_c$$

or

$$x_c \approx 747.66.$$

The total amount of sales taxes paid by the Adams is then

$$0.07 \times \$30 \times 747.66 \approx \$1,570.$$

Exercise #2. Do Problem 3.9 in the Workouts book.

(a) The slope of an indifference curve measures the MRS between grapefruits and avocado. The questions asks exactly for the (absolute value of the) MRS, which is constant in this part of the graph. Thus the answer is: she is willing to give up 2 grapefruits to get one avocado.

(b) By the same logic of part (a), she is willing to give up 1/2 grapefruit to get one avocado.

(c) The equation for the indifference curve that goes through (20,20) is

$$x_2 = \begin{cases} 30 - \frac{1}{2}x_1 \text{ for } x_2 \leq x_1 \\ 60 - 2x_1 \text{ for } x_2 \geq x_1 \end{cases}.$$

The equation for the indifference curve that goes through (10,10) is

$$x_2 = \begin{cases} 15 - \frac{1}{2}x_1 \text{ for } x_2 \leq x_1 \\ 30 - 2x_1 \text{ for } x_2 \geq x_1 \end{cases}.$$

See Figure 4 for the graph.

(d) Yes, her preferences are convex. You can check this by taking any two bundles X and Y such that that $X \sim Y$. Consider then a new bundle Z that lies on the segment that connects X and Y, and notice that

$$\begin{aligned} Z &\succeq X \\ Z &\succeq Y. \end{aligned}$$

However, these preferences are not strictly convex. Can you show it?

Exercise #3. Do Problem 4.7 in the Workouts book.

(a) The locus of points for which

$$x_1 + 2x_2 = 2x_1 + x_2$$

is also the locus of points such that

$$x_2 = x_1,$$

i.e. the $45°$ line. See Figure 5.

(b) See Figure 6. At the bundle (8,2) one sees that

$$\begin{aligned} 2x_1 + x_2 &= 18 \\ x_1 + 2x_2 &= 12. \end{aligned}$$

Therefore,

$$u(8,2) = \min\{2x_1 + x_2, x_1 + 2x_2\} = \min\{18, 12\} = 12.$$

(c) See Figure 7.

(d) The point (5,2) is below the $45°$ line. For $x_1 > x_2$ the slope of an indifference curve is -0.5 (see Figure 7). The MRS between corn chips and french fries is -0.5. The question asks for the (absolute value of the) MRS between french fries and corn chips though. The MRS between french fries and corn chips is just the inverse of -0.5, i.e., -2. The correct answer to the question is then: he would be willing to trade 2 corn chips for 1 unit of french fries.

Exercise #4. Do Problem 4.11 in the Workouts book.

(a) See Figure 8. Indifference curves are quarters of circles with center in the origin (0,0). The equation of an indifference curve reads

$$x_2 = \sqrt{\overline{u} - x_1^2}$$

where $\overline{u}$ is the utility level associated with it. One way to see if he has convex preferences is to compute the MRS and check whether its absolute value diminishes as x_1 increases. To compute the MRS, let's first compute the marginal utilities of good 1 and 2:

$$\begin{aligned} \frac{\partial U}{\partial x_1} &= 2x_1 \\ \frac{\partial U}{\partial x_2} &= 2x_2. \end{aligned}$$

Then, the MRS is

$$MRS(x_1, x_2) = -\frac{2x_1}{2x_2} = -\frac{x_1}{x_2}.$$

The absolute value of the MRS, x_1/x_2 increases as x_1 increases. Thus, these preferences are not convex.

Another way to check this is graphical (see figure 8), by drawing the segment that connects two bundles X and Y such that $X \sim Y$. You can see that Z is such that

$$\begin{aligned} X &\succ Z \\ Y &\succ Z. \end{aligned}$$

Figure 1

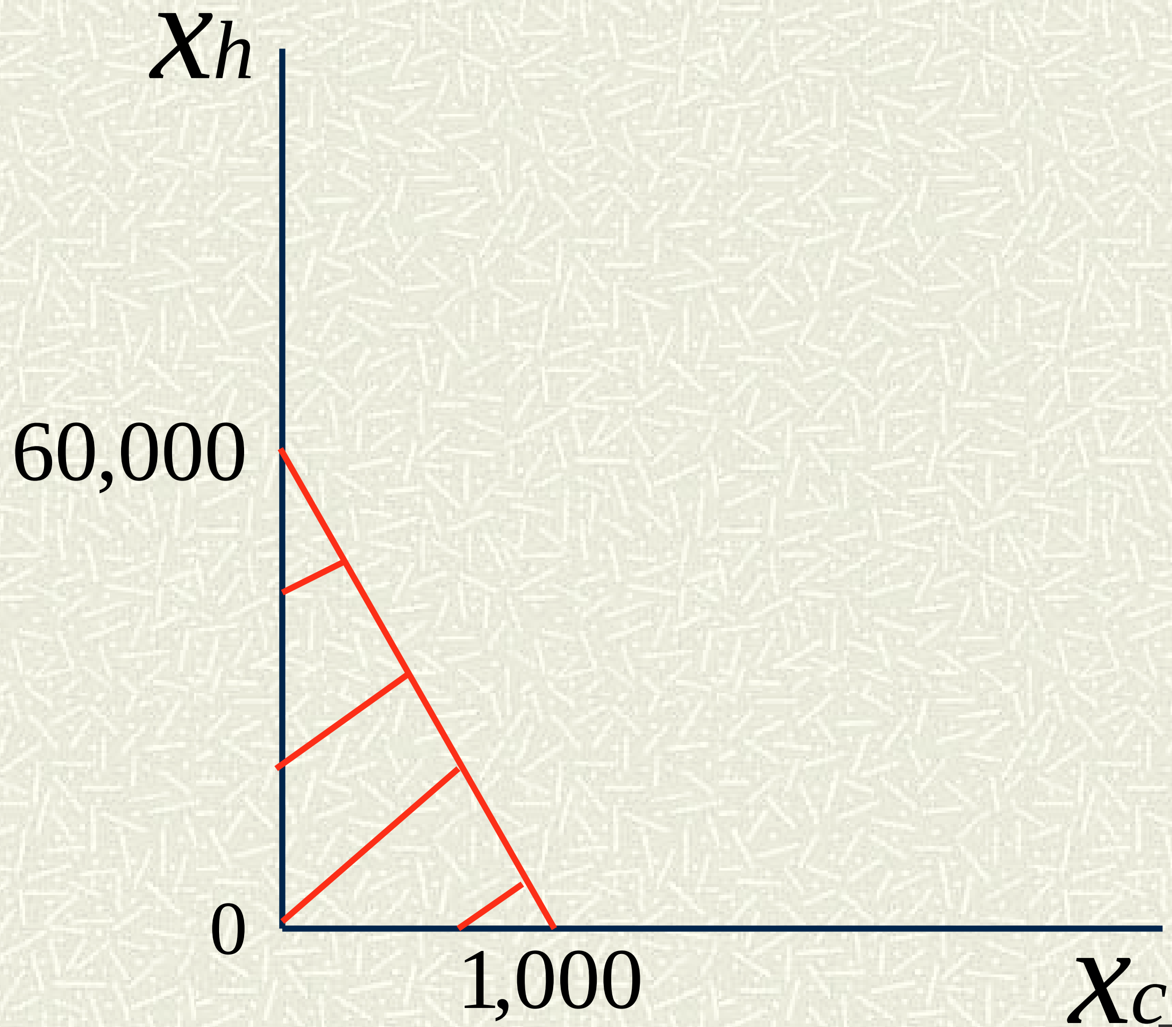

Figure 2

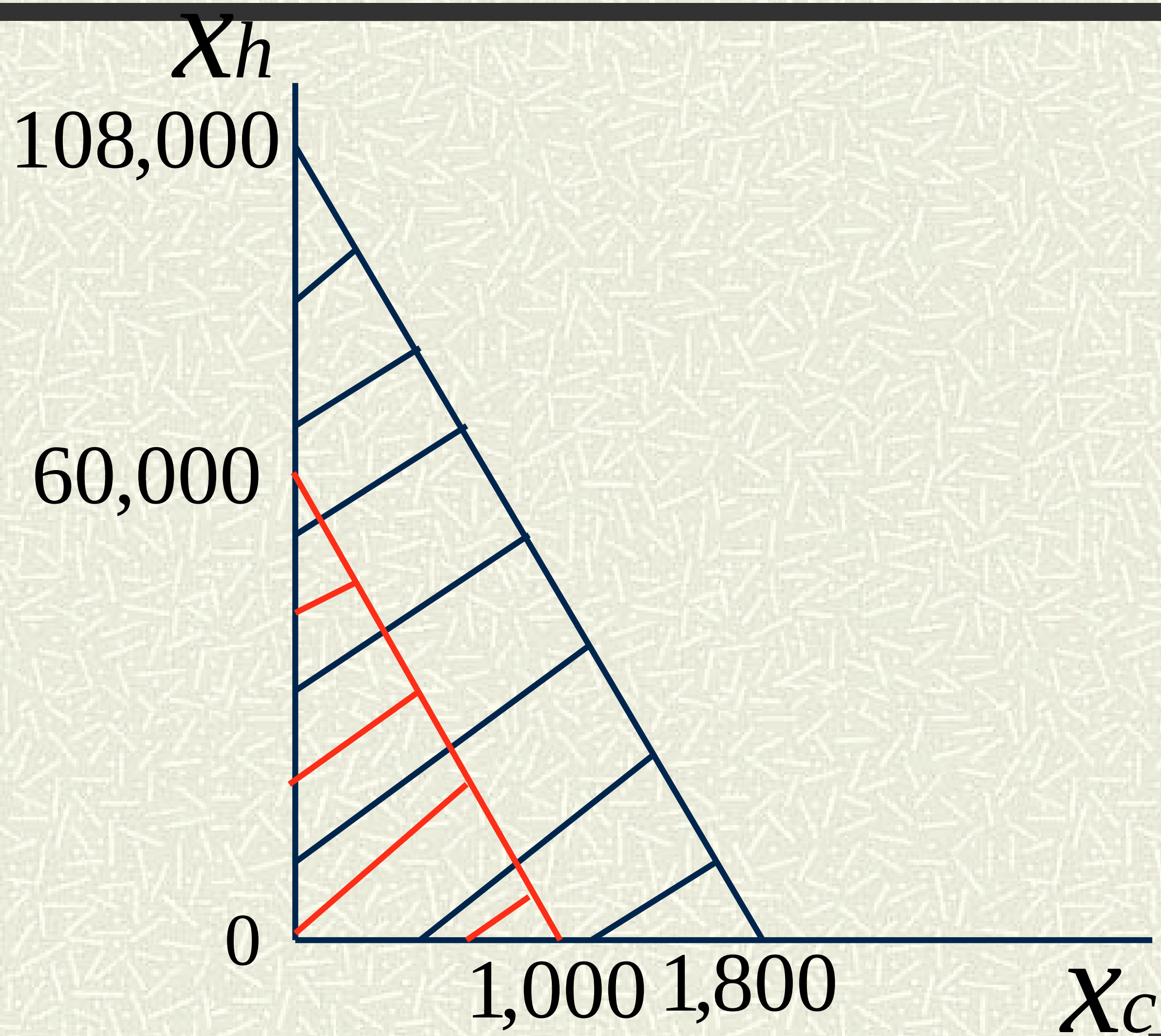

Figure 3

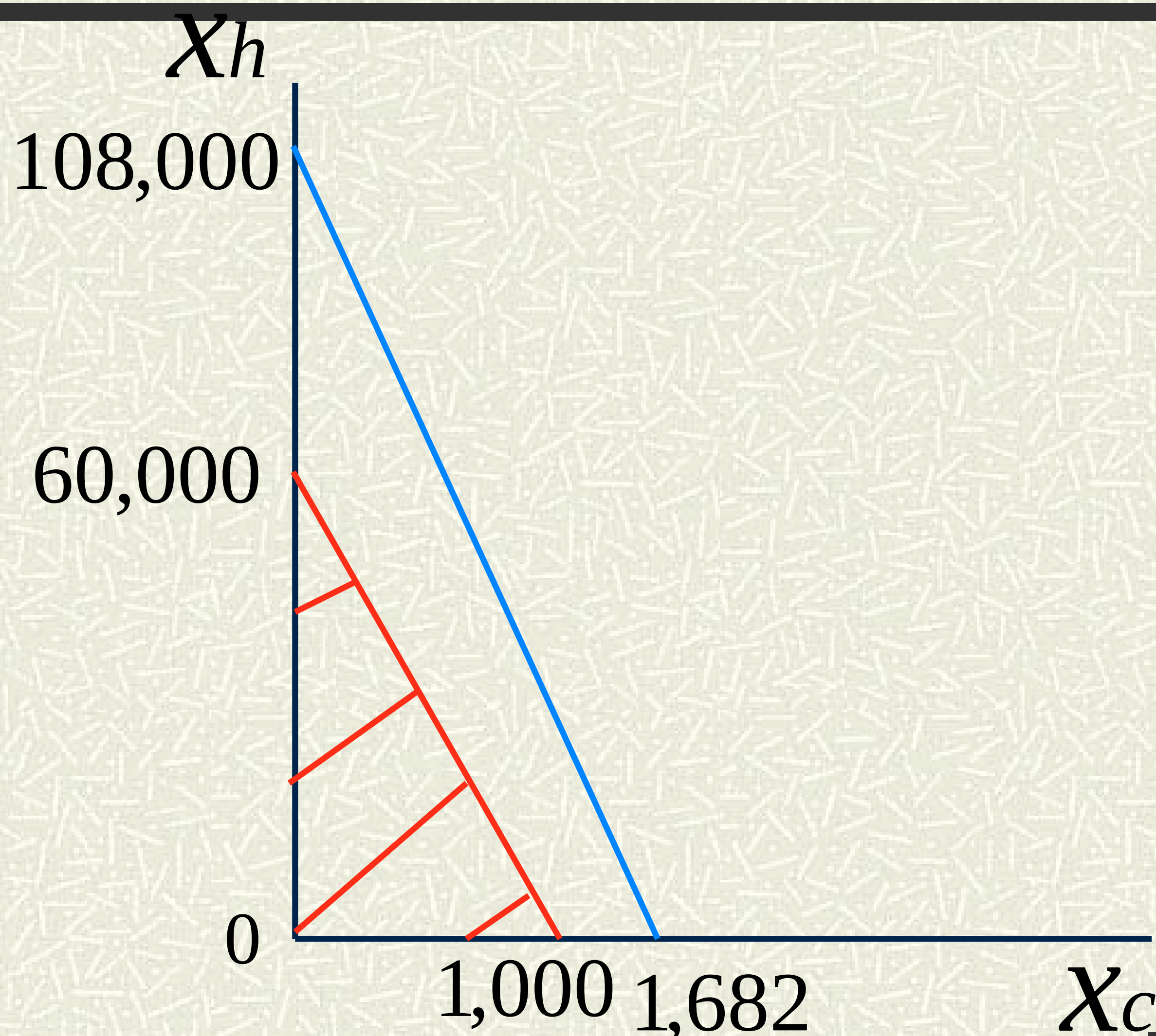

Figure 4

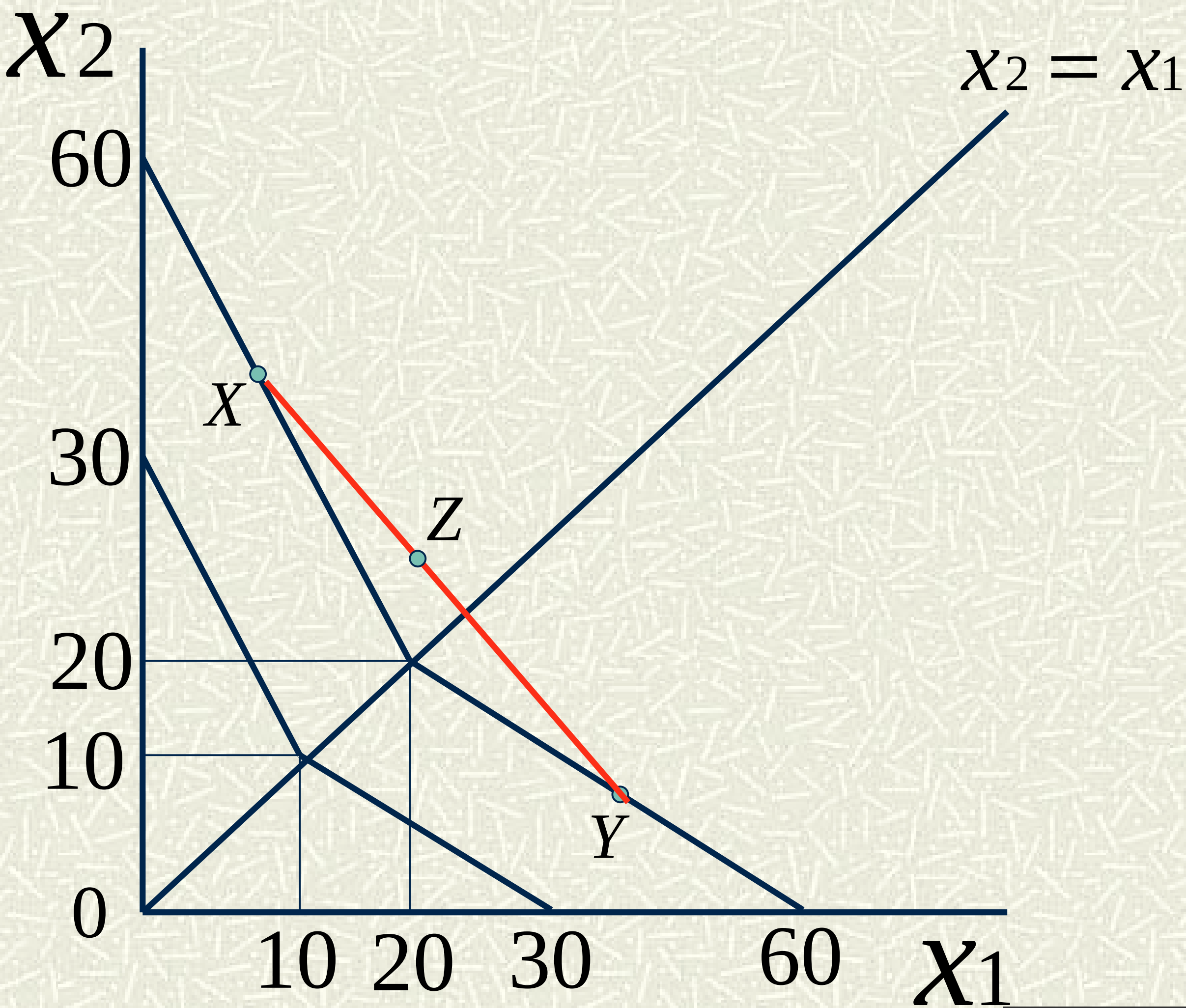

Figure 5

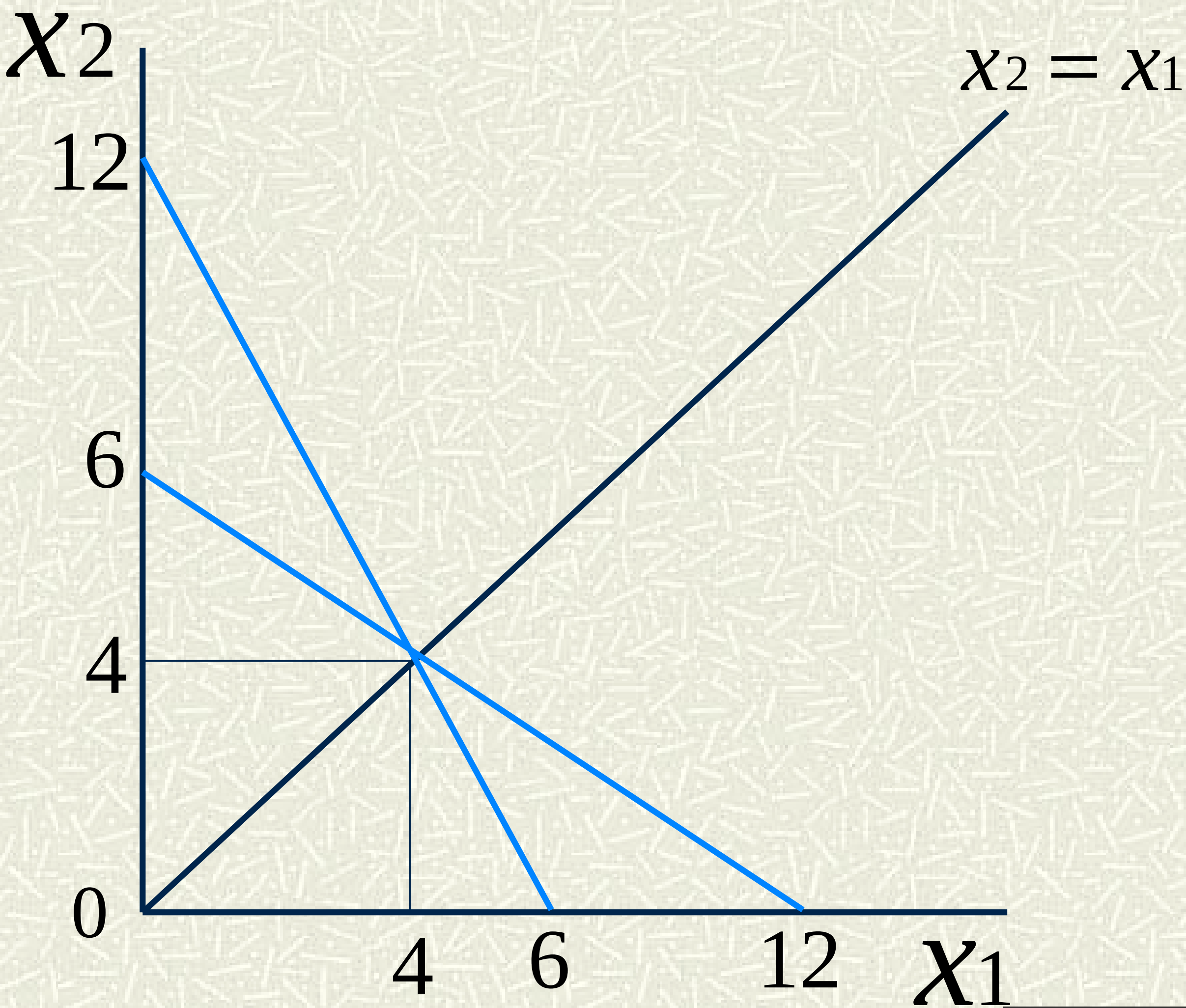

Figure 6

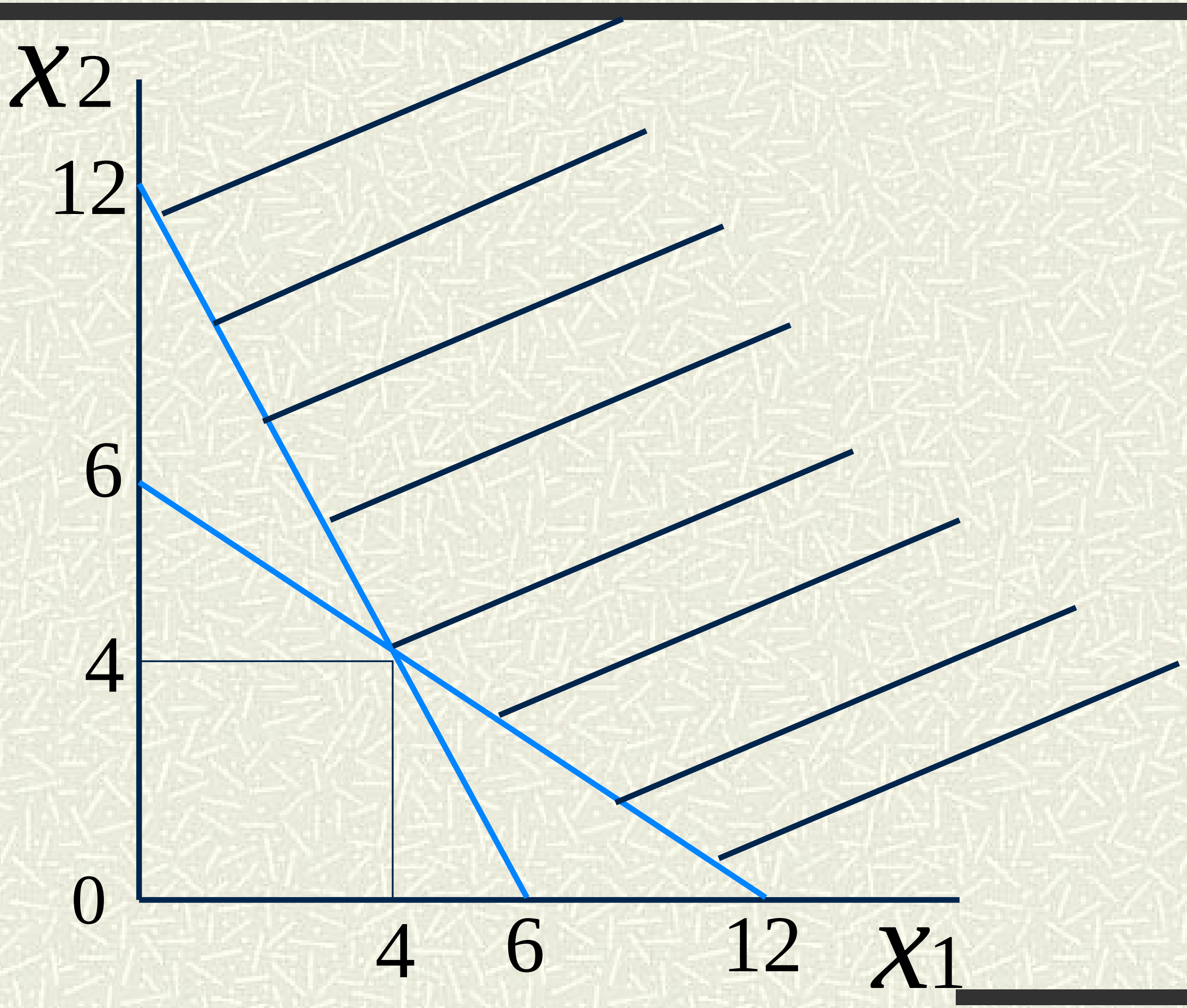

Figure 7

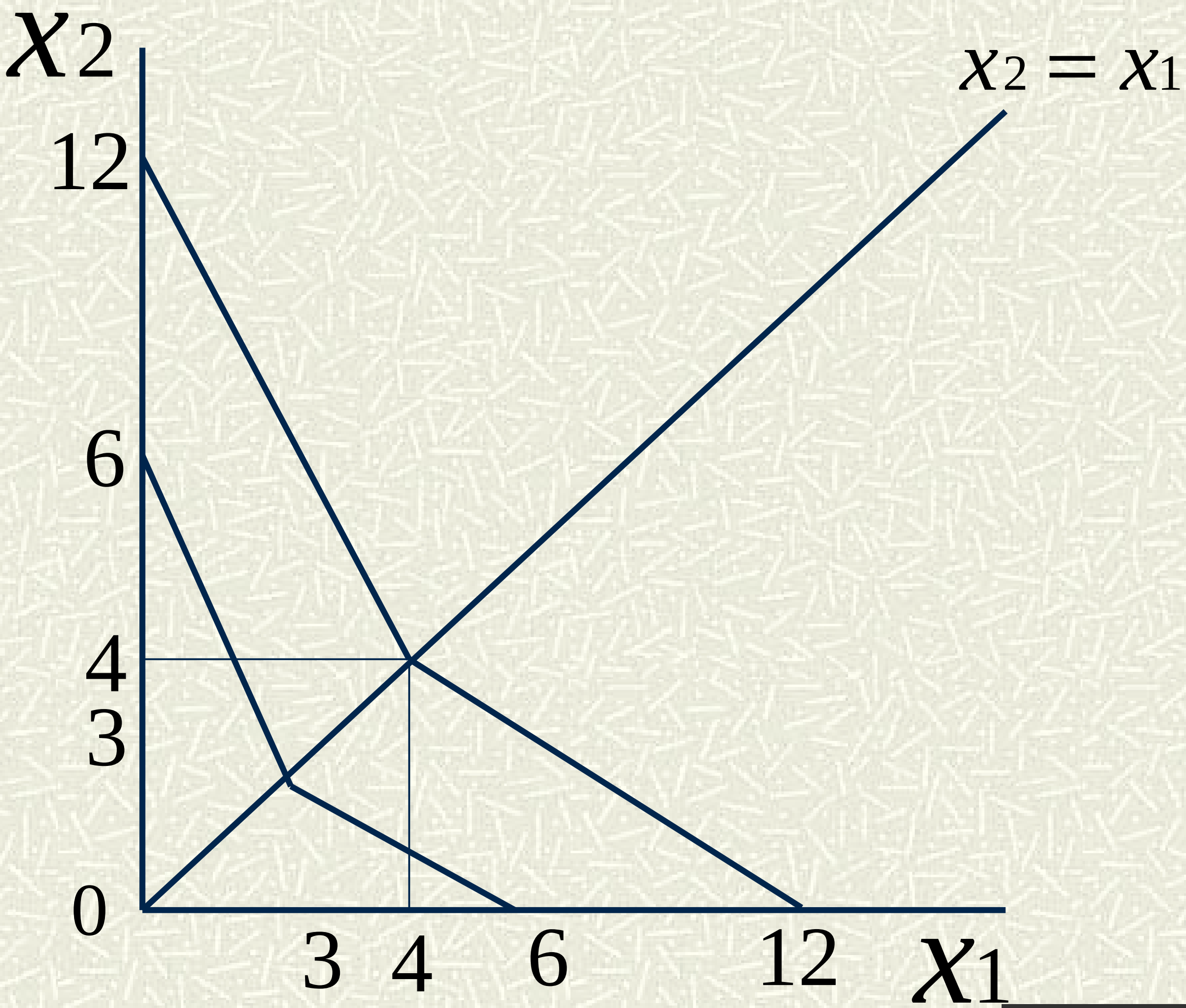

Figure 8