

Problem Set #5

This problem set is due at the beginning of your CLASS (not recitation) on Monday, April 23. There are three questions on the homework worth a total of 100 points. The points assigned to each part of each question are indicated in brackets.

Exercise #1. Let a firm's long-run cost function be

$$c(y) = \frac{y^2}{4} + F \text{ if } y > 0$$

and

$$c(y) = 0 \text{ if } y = 0.$$

(That is, the firm faces quasi-fixed costs equal to F . See p. 330 and p.353 of the textbook for a discussion of quasi-fixed costs.)

- (a) [7 pts.] What is the firm's marginal cost function? What is the firm's average cost function?
- (b) [7 pts.] At what level of output is the firm's marginal cost equal to its average cost? At what level of output is the firm's average cost minimized? Show your work. [Hint: express your answers in terms of the fixed cost F .]
- (c) [7 pts.] Assuming that the firm behaves competitively, what is the lowest price at which the firm will supply a positive amount of output? How much output would the firm supply at this price? [Hint: express your answers in terms of the fixed cost F .]
- (d) [7 pts.] Suppose $F = 25$. In a neat and clear diagram showing the firm's average and marginal cost functions, graph the firm's supply curve for output. In addition, obtain an algebraic formula for the firm's supply curve. This formula should specify how much output the firm supplies for different values of the price of output p .
- (e) [7 pts.] Now suppose that $p = 10$. Determine the firm's profit-maximizing level of output. In addition, calculate the firm's revenues, the firm's costs, and the firm's profits. Finally, in a clearly labelled diagram showing the firm's average and marginal cost curves, indicate which areas correspond to the firm's revenues, the firm's costs and the firm's profits.

Exercise #2. Consider a competitive industry consisting of a large number of identical price-taking firms, each of which has the long-run cost function

$$\begin{aligned} c(y) &= y^2 + 4 \text{ if } y > 0 \\ c(y) &= 0 \text{ if } y = 0, \end{aligned}$$

where y is the output of a typical firm. Note that each firm faces quasi-fixed costs equal to 4 (see Exercise #1). The market demand curve in this industry is described by the equation

$$Q_D(p) = 400 - 4p,$$

where $Q_D(p)$ is the quantity demanded when the market price is p .

(a) [7 pts.] Derive the equation of the supply curve of a typical firm (i.e. express the optimal output of a typical firm as a function of the price p).

(b) [7 pts.] Suppose that there are N firms in the industry. Use your answer from part (a) to derive the equation of the industry supply curve (i.e. express total industry supply as a function of N and p).

(c) [7 pts.] Determine the long-run equilibrium number of firms in this industry. What is the equilibrium price of the good? How much output does a typical firm produce? What is the total amount of output produced in this market? What are the profits earned by a typical firm? Show your work.

(d) [7 pts.] Suppose now that the market curve shifts outwards. In particular, the equation of the new market demand curve is

$$Q_D(p) = 600 - 4p.$$

Assuming that the number of firms remains fixed (in the short-run) at the value determined in part (c), compute the new equilibrium price. How much output does a typical firm produce? What is the total amount of output produced in the market? What are the profits earned by a typical firm?

(e) [7 pts.] After the shift in the market demand curve, what is the new long-run equilibrium number of firms in this industry? How many new firms enter the industry as a result of the increase in demand?

Exercise #3. This problem examines the behavior of the fishing industry in the fishing grounds off the coast of Newfoundland. Suppose that a typical fishing boat can catch $f(B)$ dollars' worth of fish in a typical year, where B is the total number of fishing boats operating in the fishing grounds. Specifically, let

$$f(B) = \frac{1,200,000}{B^{1/2}},$$

so that the value of the fish caught by a typical boat decreases as the number of boats operating in the fishing grounds increases. Let the annual cost of operating a fishing boat be \$60,000.

(a) [3 pts.] Suppose that $B = 100$. What are the annual profits (i.e. value of fish caught minus operating cost) of a typical fishing boat?

(b) [5 pts.] Let the annual interest rate be 10%. Assuming that a typical fishing boat stays in business indefinitely, and that the total number of fishing boats remains unchanged at 100, what is the net present value of a typical fishing boat's stream of profits? (Note: the net present value of an infinite sequence of constant annual payments is A/R , where A is the annual payment and R is the annual interest rate.)

(c) [**5 pts.**] Suppose that there is free entry and exit into the fishing industry. How many fishing boats will operate in equilibrium? Explain your reasoning. (Hint: In an equilibrium with free entry and exit, a typical fishing boat must make zero profits.)

(d) [**7 pts.**] The government in Newfoundland decides to issue 144 fishing licenses. Each boat operating off the coast of Newfoundland must purchase one of these licenses. The government allows these licenses to be bought and sold in a competitive market. Assuming that the annual interest rate is 10%, what is the market price of a fishing license? (Hint: In equilibrium, a typical fishing boat must earn zero profits, net of the cost of acquiring a fishing license. Thus, the cost of a fishing license must be equal to the net present value of the profit stream generated by a typical fishing boat.)

(e) [**2 pts.**] Suppose now that the government issues 225 fishing licenses. What is the market price of a fishing license in this case?

(f) [**8 pts.**] Finally, suppose that the government issues 625 fishing licenses. What is the market price of a fishing license in this case? (Hint: if 625 boats fish, the profits of a boat will be negative, which cannot be an equilibrium in the long-run.)