Econ 73-250A-F
Spring 2001
Prof. Daniele Coen-Pirani

Suggested Answers for the Final Exam

Exercise #1. In the short-run the number of firms is constant at 84. The supply function of each firm is found by setting $p = MC(y)$, and taking into account the possibility that a firm can always shut-down and make zero profits. The condition $p = MC(y)$ yields

$$y = \frac{p}{8}.$$

A firm makes positive profits only if the output price is at least 16. Therefore a firm's supply curve is

$$y = \begin{cases} \frac{p}{8} \text{ if } p \geq 16 \\ 0 \text{ if } p \leq 16 \end{cases}.$$

Let p_s denote the price suppliers get and p_d the price consumers pay. After the introduction of the subsidy we have

$$p_s = p_d + 2.$$

Demand is therefore

$$Q_D(p_d) = 200 - 2p_d,$$

while supply is

$$Q_S(p_s) = \frac{84}{8} p_s.$$

Equilibrium requires

$$Q_D(p_d) = Q_S(p_d + 2),$$

or

$$\begin{aligned} 200 - 2p_d &= \frac{84}{8}(p_d + 2) \\ 200 - 2p_d &= \frac{84}{8} p_d + \frac{84}{4} \\ p_d &= \frac{179}{12.5} \approx 14.32 \\ p_s &\approx 16.32. \end{aligned}$$

Notice that you could have replaced the supply price into the equilibrium condition, so that instead of

$$Q_D(p_d) = Q_S(p_d + 2),$$

you would have imposed

$$Q_D(p_s - 2) = Q_S(p_s).$$

Solving the problem in this way gives rise to the same answer because $p_s = p_d + 2$.

Exercise #2. The doctor is able to extract all the surplus from consumers. He will sell visits until the marginal cost of providing a visit, 10, equals the price the patients are willing to pay for it:

$$10 = 50 - \frac{q}{2}$$

which yields $q^* = 80$. Each visit is sold at a different price, i.e., exactly the price consumers are willing to pay for it. The doctor's profits are given by the consumer's surplus minus total costs:

$$(50 - 10)\,\frac{80}{2} = 1{,}600.$$

See Figure 1.

Exercise #3. The emissions trading approach allows the government to achieve a pollution target in a more cost effective manner than direct regulation of emissions. This is because the latter approach cannot work well if the regulator does not have access to information on the marginal cost functions of reducing emissions for different plants. This condition is unlikely to be verified. By setting up a market for emissions the government can in principle achieve the same objective as a regulator that has access to the information mentioned above, without actually needing to have that information. In the emissions market firms with a relatively high marginal cost of emissions reduction can buy emissions permits from firms whose marginal costs are relatively low. Therefore the "low marginal cost" firms will be the ones that will reduce emissions most, an objective that a cost effective approach should achieve.

Exercise #4. The deadweight loss is found by comparing total surplus under the monopolist with total surplus when the efficient amount of output is produced. The efficient amount of output is produced when

$$p = MC(y),$$

where $p = 100 - y$. Therefore

$$100 - y = 10$$

or $y^* = 90$. Total surplus when the efficient amount of output is produced is

$$\frac{90\,(100 - 10)}{2} = 4{,}050.$$

The monopolist will produce a quantity y so that

$$MR(y) = MC(y)$$

or

$$100 - 2y = 10$$

which gives $y^* = 45$. The price is $100 - 45 = 55$. Therefore consumer's surplus is

$$\frac{(100 - 55)\,45}{2} = 1{,}012.5$$

and producer's surplus is

$$(55 - 10)\,45 = 2{,}025.$$

Total surplus is therefore 3,037.5. The deadweight loss is $4{,}050 - 3,037.5 = 1{,}012.5$. See Figure 2.

Exercise #5. The firm solves the following problem

$$\max_{x_1,x_2} 4x_1^{\frac{1}{2}}x_2^{\frac{1}{4}} - 2x_1 - x_2.$$

To solve for the quantities, write the first order conditions (output price times marginal product of a factor equals rental price of that factor)

$$\begin{aligned} \frac{4}{2}x_1^{\frac{1}{2}-1}x_2^{\frac{1}{4}} - 2 &= 0 \\ \frac{4}{4}x_1^{\frac{1}{2}}x_2^{\frac{1}{4}-1} - 1 &= 0. \end{aligned}$$

These equations can be rewritten as

$$\begin{aligned} x_1^{-\frac{1}{2}}x_2^{\frac{1}{4}} &= 1 \\ x_1^{\frac{1}{2}}x_2^{-\frac{3}{4}} &= 1. \end{aligned}$$

It is easy to see how this system is solved by $x_1^* = 1$ and $x_2^* = 1$ (just replace these numbers in the equations above).

Exercise #6. To answer the question we need to solve for the demand function for good x. This is achieved by solving the following problem:

$$\begin{aligned} &\max_{x,y} x - \frac{x^2}{2} + y, \\ &\text{s.t.} \\ 300 &= p_x x + y. \end{aligned}$$

The first order condition is

$$MRS(x,y) = -\frac{p_x}{p_y},$$

where

$$MRS(x,y) = -\frac{1-x}{1} = 1 - x.$$

Replacing, we get

$$1 - x = \frac{p_x}{p_y}, \tag{1}$$

or

$$x^* = 1 - \frac{p_x}{p_y}.$$

Use the budget line to get

$$\begin{aligned} y^* &= 300 - p_x x^* \\ &= 300 - p_x \left(1 - \frac{p_x}{p_y}\right). \end{aligned}$$

If $p_x = 0.5$ and $p_y = 1$, then $x^* = 0.5$. The consumer's surplus associated with this choice is found by computing the area below the inverse demand curve for good x and above the price p_x:

$$0.5 \frac{(1 - 0.5)}{2} = 0.125.$$

If $p_x = 1$, $x^* = 0$. The consumer's surplus associated with this choice is zero. Therefore the change in consumer's surplus is -0.125. See Figure 3.

Another way to answer this question is to compute the change in utility for the consumer:

$$u(0, 300) - u(0.5, 299.75) = -0.125.$$

Notice: there is no surplus associated with consuming good y because utility is linear in y. The consumer pays for y exactly the amount he is willing to pay. To see this notice that from the first order condition (1)

$$p_y = \frac{p_x}{1 - x}.$$

This equation tells us how much the consumer is willing to pay for an extra unit of good y. This amount is constant and independent of y (because utility is linear in y). Since the amount the consumer is willing to pay does not depends on y, it must coincide with the price the consumer is paying for each unit (if it is greater, the consumer would demand an infinite amount of the good, while if it is lower the consumer would demand zero units). Even if by consuming less of good x the consumer ends up consuming more of good y, he does not get any extra surplus from y. Thus, it is enough to compute the change in the area below the demand curve to find the total change in surplus.

FIGURE 1

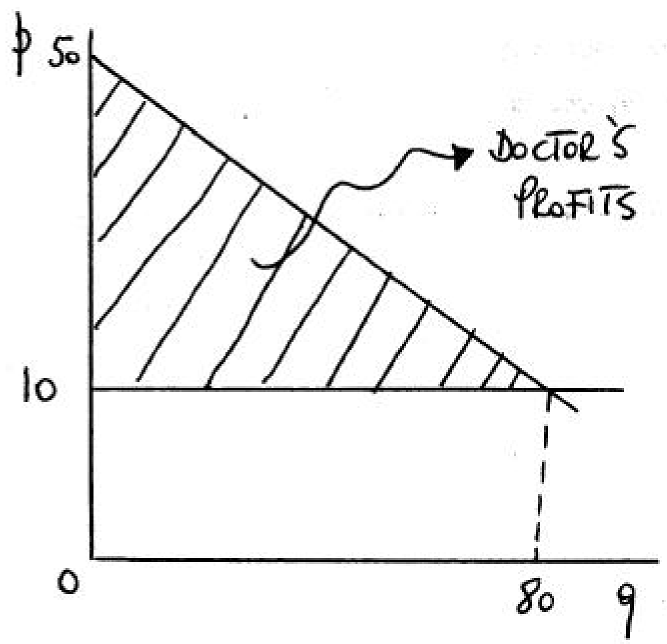

FIGURE 2

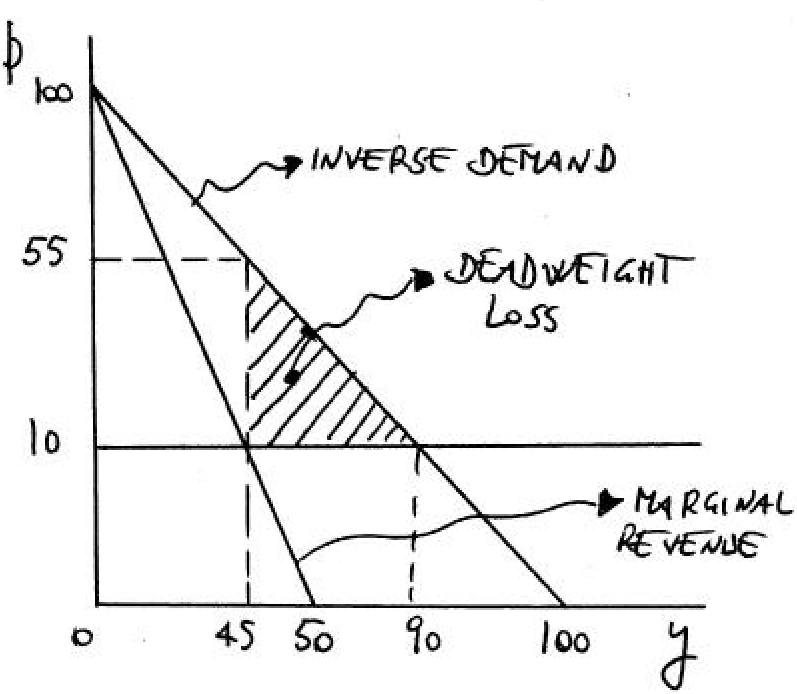

FIGURE 3