Econ 73-250A-F
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Suggested answers to Midterm Examination #2

Please refer to the end of this document for all graphs.

Exercise #1. (a) The equilibrium price and quantity can be found by setting quantity supplied equal to quantity demanded:

$$Q_S(p) = Q_D(p),$$

or substituting

$$600 = 1{,}000 - p.$$

The equilibrium price is then

$$p^* = 400.$$

The equilibrium quantity is of course 600.

(b) There are two equilibrium prices: the price paid by the students out of their pockets and the price received by the owners. Let p_D denote the first and p_S the second. We know that

$$p_S = p_D + 100.$$

The quantity demanded is a function of p_D:

$$Q_D(p_D) = 1{,}000 - p_D,$$

while the quantity supplied, which in principle is a function of p_S, is in this case constant:

$$Q_S(p_S) = 600.$$

In equilibrium then

$$Q_S(p_S) = Q_D(p_D),$$

or substituting

$$1{,}000 - p_D = 600.$$

Thus, the students keep paying out of their pockets the same amount as before the subsidy:

$$p_D^* = 400.$$

The owners benefit from the policy because the equilibrium rent increases:

$$p_S^* = 400 + 100 = 500.$$

(c) See Figure 1.

(d) The consumer's surplus does not change, as consumers pay the same amount as before the subsidy and rent the same number of apartments. The supplier's surplus increases: now the owners of rented apartments get \$500 a month of rent. They still rent 600 apartments in a year. Their surplus increases by $\$(500-400)600 = \$60{,}000$ a year. The cost for CMU of this policy is $\$100(600) = 60{,}000$ a year. Thus, the policy only transfers money from CMU to the owners of houses for rent. There is no deadweight loss.

Exercise #2. (a) The firm's technology does not display increasing returns to scale, but rather constant returns to scale. In fact, for $t > 1$:

$$y_t = \min\{2tK, tL\} = t\min\{2K, L\} = ty$$

where

$$y = \min\{2K, L\}.$$

(b) See Figure 2.

(c) To determine the cost function, solve the problem

$$\begin{aligned} & \min_{K,L}\{4L + 16K\} \\ & s.t. \\ y &= \min\{2K, L\}. \end{aligned}$$

At an optimal choice the firm is going to choose inputs in such a way that

$$2K = L.$$

Thus, from the production function $y = 2K = L$:

$$\begin{aligned} K &= \frac{y}{2} \\ L &= y. \end{aligned}$$

Replace into the objective function to get the cost function

$$c(y) = 4y + \frac{16}{2}y = 12y.$$

(d) See Figure 3. The average cost function coincides with the marginal

$$\begin{aligned} AC(y) &= \frac{c(y)}{y} = \frac{12y}{y} = 12, \\ MC(y) &= \frac{\partial c(y)}{\partial y} = 12. \end{aligned}$$

Exercise #2. (a) If the price of the fixed factor is increased profits will decrease. In fact:

$$\Pi = py - c_v(y) - F,$$

where p is the output price, y is output, $c_v(y)$ are variable costs, and F represents fixed costs.

If the price of the fixed factor increases, i.e., F increases, Π decreases.

(b) The firm should increase the amount of factor 1 in order to increase profits. In fact, by increasing input 1 by one unit, revenue goes up by pMP_1, while cost goes up by w_1, with

$$pMP_1 > w_1.$$

(c) Profits would more than double. Consider a firm producing output using capital and labor with an increasing returns to scale technology $f(K, L)$ such that

$$f(2K, 2L) > 2f(K, L).$$

This firm's profits are:

$$\Pi = pf(K, L) - w_k K - w_l L.$$

After doubling all inputs

$$\begin{aligned} \Pi_2 &= pf(2K, 2L) - w_k(2K) - w_l(2L) \\ &> p2f(K, L) - w_k(2K) - w_l(2L) \\ &= 2[pf(K, L) - w_k K - w_l L] \\ &= 2\Pi. \end{aligned}$$

Thus, profits more than double:

$$\Pi_2 > 2\Pi.$$

FIGURE 1

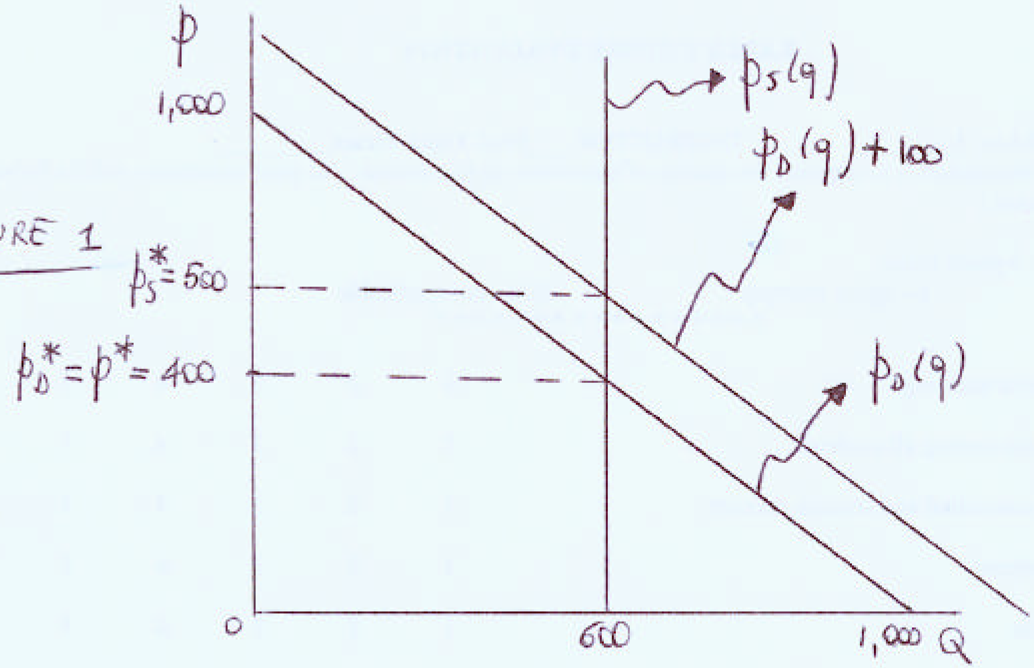

FIGURE 2

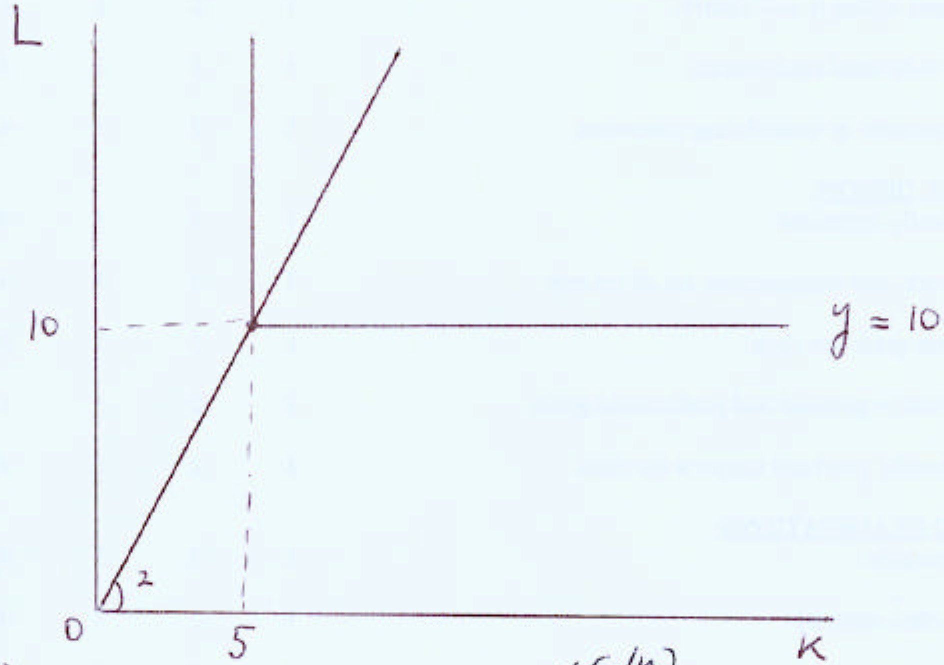

FIGURE 3