Econ 73-250A-F
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Prof. Daniele Coen-Pirani

Answers to midterm examination #1

Please refer to the end of this document for all graphs.

Exercise #1. (a) For this question you could have proceeded in different ways:

One way. Maximize a Cobb-Douglas utility function subject to the budget line:

$$\max_{x_m, x_c} \left(x_m^{0.1} x_c^{0.9} \right) \quad \text{subject to} \quad 400 = x_c + x_m.$$

First substitute the budget line in the utility function by replacing one of the variables, say x_c, to get

$$\max_{x_m} \left[x_m^{0.1} (400 - x_m)^{0.9} \right].$$

The first-order condition for this maximization problem is

$$0.1 x_m^{0.1-1} (400 - x_m)^{0.9} - 0.9 x_m^{0.1} (400 - x_m)^{0.9-1} = 0.$$

This can be simplified to yield:

$$0.1 x_m^{-1} = 0.9 (400 - x_m)^{-1}.$$

Further simplification yields:

$$0.1 (400 - x_m) = 0.9 x_m \Rightarrow x_m^* = 40.$$

Plugging this number in the equation for the budget line we get

$$x_c^* = 400 - x_m^* = 360.$$

See Figure 1 for the graph.

Another way. Another way to go is to recognize that Cobb-Douglas preferences satisfy all the assumptions we have mentioned in class (no boundary solutions, no kinks in indifference curves, convexity) that are necessary and sufficient for the optimal behavior of the consumer to be captured by the equality between the MRS and (minus) the price ratio

$$MRS(x_m^*, x_c^*) = -\frac{p_m}{p_c}.$$

Notice that this condition is equivalent to the first-order condition derived in the previous point. The marginal rate of substitution between x_m and x_c is just

$$MRS(x_m, x_c) = -\frac{0.1}{0.9} \frac{x_c}{x_m}.$$

The ratio of the prices is just 1. Thus, at the optimal point:

$$-\frac{0.1}{0.9}\frac{x_c}{x_m} = -1.$$

Now you can replace the budget line into this expression to get rid of one of the variables. For example, by replacing x_c we get

$$\frac{0.1}{0.9}\frac{(400 - x_m)}{x_m} = 1.$$

Rearranging:

$$0.1\,(400 - x_m) = 0.9x_m \Rightarrow x_m^* = 40.$$

Plugging this number in the equation for the budget line we get

$$x_c^* = 400 - x_m^* = 360.$$

See Figure 1 for the graph.

(b) The equation for the budget line is

$$\begin{aligned} 0.5x_m + x_c &= 400 \text{ if } x_m \leq 50, \\ x_m + x_c &= 425 \text{ if } x_m > 50. \end{aligned}$$

It is possible to find this line by means of the following argument. If Anna does not buy any milk, then she can spend \$400 on other things. The point (0,400) is therefore a point on the budget line. For each gallon of milk that Anna buys to a maximum of 50, she pays only 50 cents. If she consumes 50 gallons of milk, she pays \$25 for them, and she is left with \$375 to spend on the composite good. In other words, the point (50,375) is also a point on the budget line. In addition, all of the points on the line connecting (0,400) and (50,375) are on the budget line. The slope of this line segment is -1/2: for each extra gallon of milk, Anna has to reduce her expenditures on other things by 50 cents.

Once Anna reaches the point (50,375), the slope of her budget line changes to -1 (i.e. the slope of the budget line in part (a)). This is because Anna does not receive discounts on gallons of milk beyond the 50-th. Finally, if Anna spends all her income on milk, she can buy 425 gallons of milk: in this case, Anna spends \$25 for the first 50 gallons, and the remaining \$375 for the other 375 gallons (since the price is \$1 after the 50-th gallon). All the points on the line connecting (50,375) and (425,0) are therefore on the budget line. The coordinates of the kink point are (50,375).

See Figure 2 for the graph.

(c) We need to show how to compute a compensating variation. The compensating variation is the amount of money - call it cv - that must be given to Anna after the price increase so that her utility is the same as before the price increase.

When the milk price is \$1 her utility is given by (see point a):

$$u\,(40{,}360) = 40^{0.1}360^{0.9}.$$

To compute Anna's utility when the milk price is \$2 and she receives the transfer cv, we must find how much milk and composite good she buys in those circumstances. In other words, we need to solve the optimization problem

$$\max_{x_m, x_c} \left(x_m^{0.1} x_c^{0.9} \right) \text{ subject to } 400 + cv = 2x_m + x_c.$$

The optimal amounts $x_c^*(cv)$ and $x_m^*(cv)$ that Anna chooses of course depend on cv. Her utility will also depend on cv:

$$u(x_m^*(cv), x_c^*(cv)) = (x_m^*(cv))^{0.1} (x_c^*(cv))^{0.9}.$$

Now, to find cv set

$$u(40, 360) = u(x_m^*(cv), x_c^*(cv)).$$

This is one equation in one unknown (cv), which can be solved for cv.

Exercise #2. (a) Beer is an ordinary good because its demand increases when its price decreases.

(b) Beer is a substitute for wine because as the price of wine goes up, demand for beer increases.

(c) The relative price of a gallon of beer in terms of bottles of wine is

$$\frac{p_b}{p_w} = \frac{\$15}{\$10} = 1.5.$$

(d) The demand function for beer is:

$$x_b = 110 - 2p_b.$$

To plot the demand curve we need to compute the inverse demand function:

$$p_b = 55 - \frac{1}{2} x_b.$$

See Figure 2 for a plot of this function. The loss in consumer's surplus is given by the sum of the shaded areas of the rectangle and the triangle in figure 2: the area of the rectangle is $70(5) = 350$. The area of the triangle is given by $(80 - 70)5/2 = 25$. Summing up these two areas we get that the loss in consumer's surplus is \$375.

(e) The rectangular subregion represents the loss of surplus on the gallons that John now buys at a higher price. The triangular subregion represents the loss in surplus due to the fact that John reduces his demand for beer after the price increase.

Exercise #3. (a) TRUE. See Figure 3. In this figure, and in this discussion I am assuming that the tax revenues are kept the same across these two tax experiments [If tax revenues were not the same of course this statement could easily be proved false.] The intuition here is that, when preferences are of the perfect complement type, the indifference curves display a kink at the bundle where the consumer is making the optimal choice. Thus, there is no tangency between the budget line and the marginal rate of substitution at the optimal point. This tangency condition was responsible for the loss in consumer's utility in the example we saw in class where indifference curves were smooth. In that example, from the tangency condition

$$MRS(x_1, x_2) = -\frac{p_1 + t}{p_2}$$

we could see that the presence of the tax gave incentives to the consumer to decrease his consumption of good 1 with respect to the situation where income was taxed. When income is taxed the optimality condition reads:

$$MRS(x_1, x_2) = -\frac{p_1}{p_2}$$

and the tax does not distort the consumer's choice between the two goods.

(b) FALSE. All our assumptions on preferences cannot rule out the existence of Giffen goods, i.e., goods whose demand increase as their price increases.

(c) FALSE. The marginal rate of substitution measures the rate at which the consumer, and **not** the market, is willing to substitute one good for the other.

(d) FALSE. An indifference curve represents the collection of all bundles among which the consumer is indifferent.

(e) FALSE. A lump sum subsidy to a consumer does not affect the relative price of the goods the consumer is buying. Therefore it does not affect the optimality condition $MRS = -p_1/p_2$. However, it **does** affect the consumer's behavior because after the subsidy the consumer is going to buy more of at least one of the goods under consideration (because he has more money to spend, or some food coupons as in the Food Stamp program).

FIGURE 1

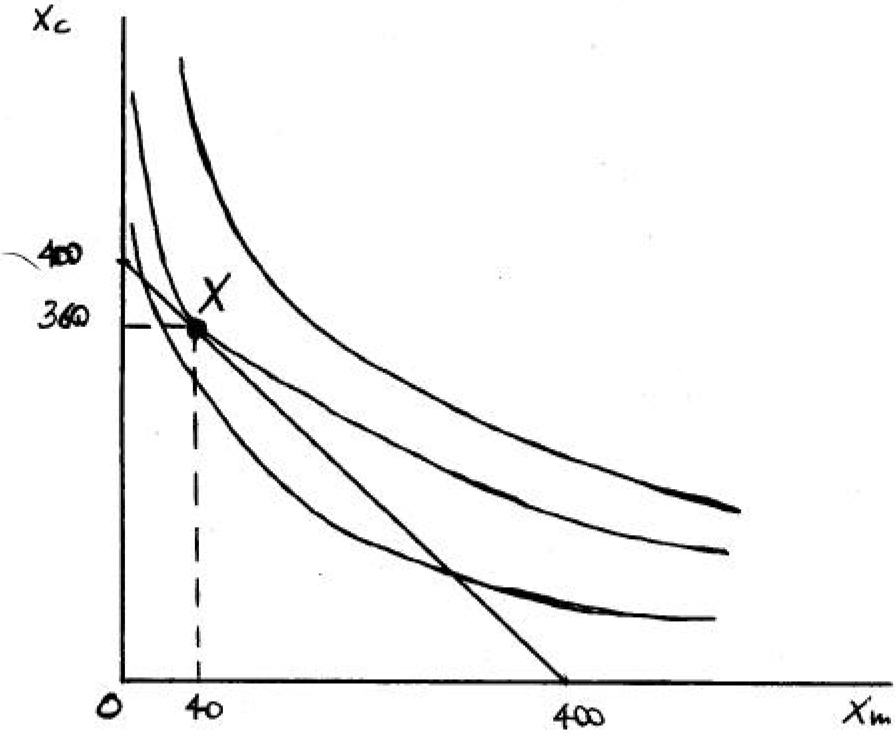
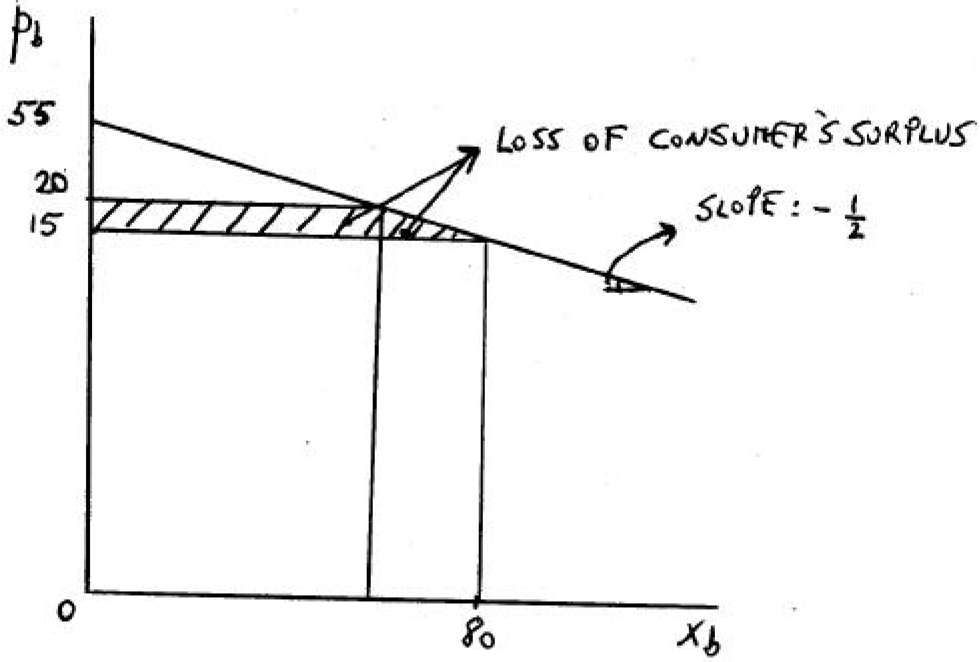
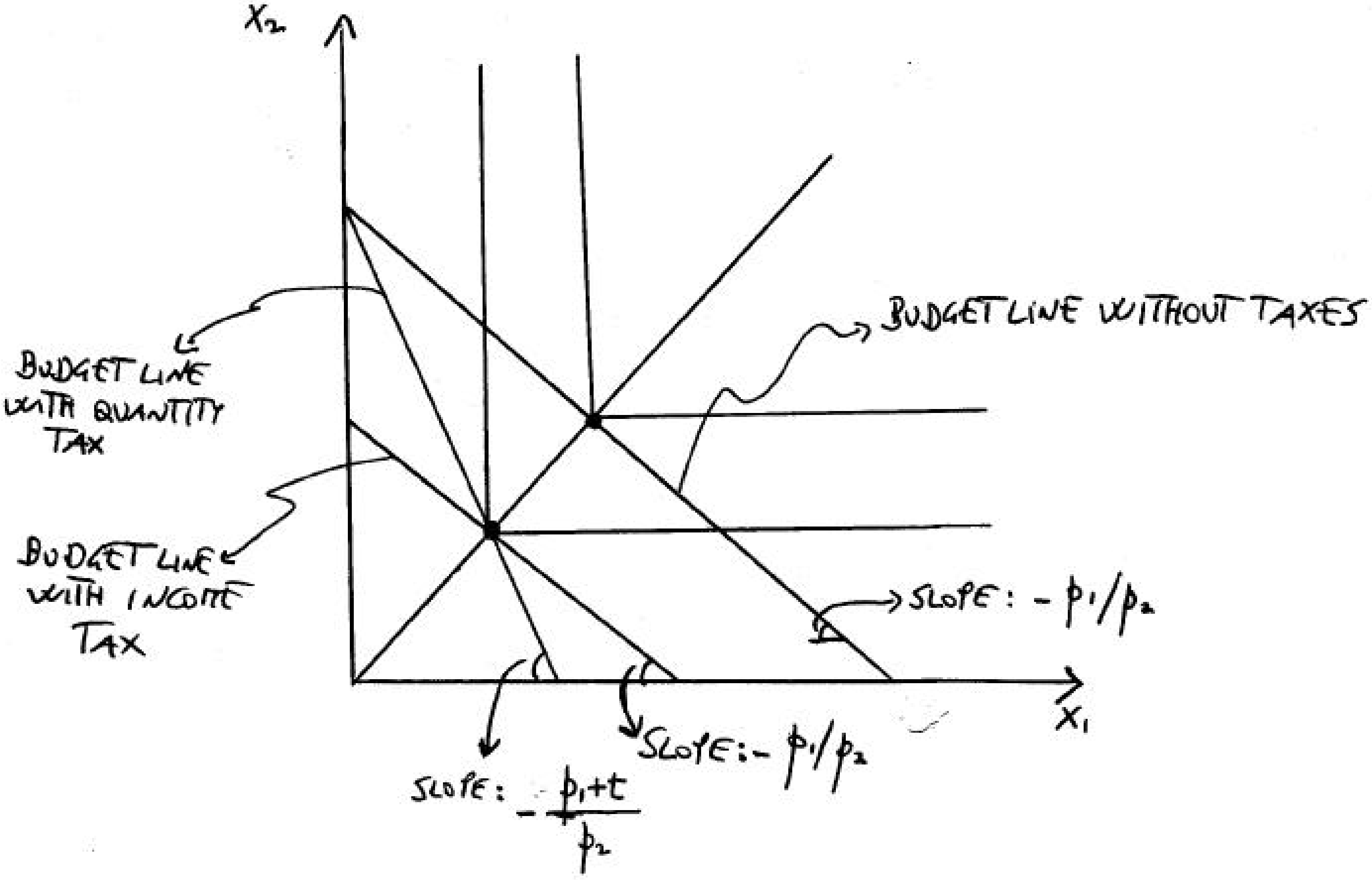


FIGURE 2





X_2
BUDGET LINE WITHOUT TAXES
BUDGET LINE WITH QUANTITY TAX
BUDGET LINE WITH INCOME TAX
SLOPE: $-p_1/p_2$
X_1
SLOPE: $-p_1/p_2$
SLOPE: $-\frac{p_1+t}{p_2}$