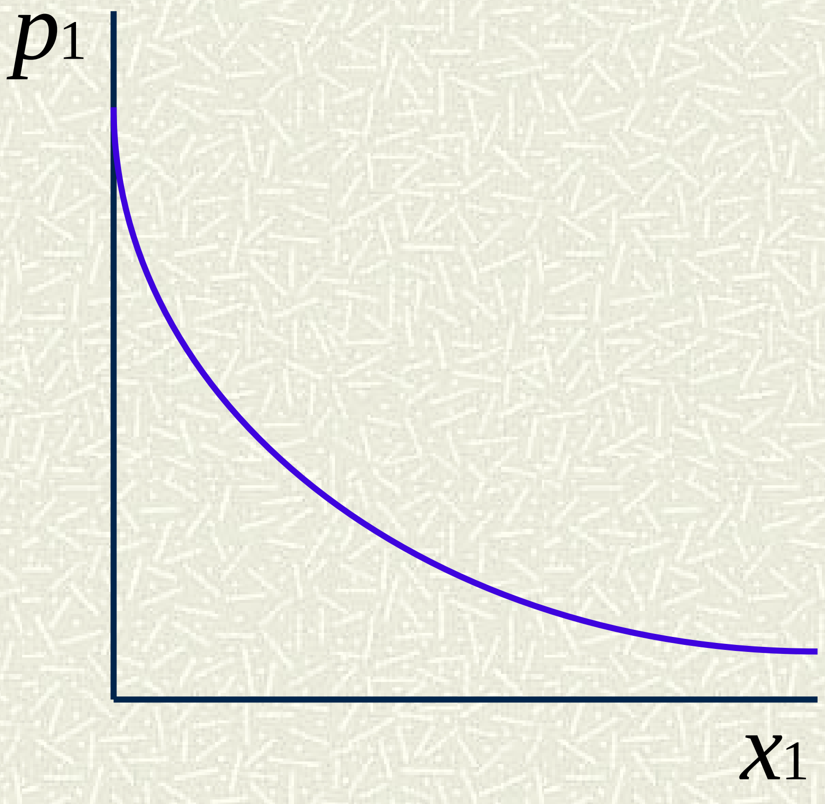
Consumer Surplus

Demand Function and Demand Curve

- Demand function:

$$x_1 = x_1(p_1, p_2, m)$$

- Demand Curve:



Inverse Demand Function

- Consider a demand function

$$x_1 = x_1(p_1, p_2, m)$$

- The inverse demand function is

$$p_1 = p_1(x_1)$$

Cobb-Douglas example:

$$x_1 = c\frac{m}{p_1} \longrightarrow$$

$$p_1 = c\frac{m}{x_1}$$

Inverse Demand Curve

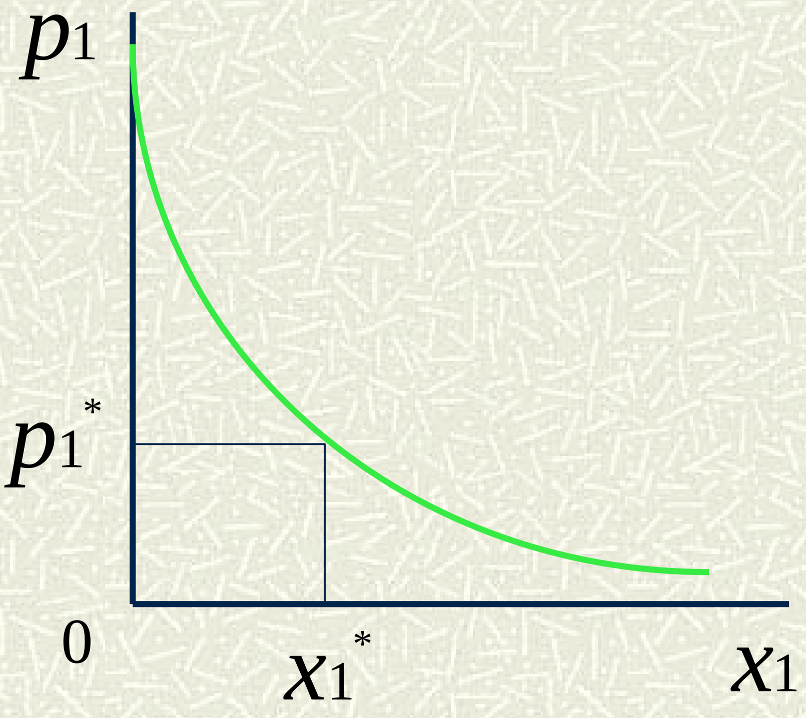
- Optimal choice:

$$\frac{p_1}{p_2} = -MRS$$

- Suppose: $p_2 = 1$ (composite good)
- Rearrange:

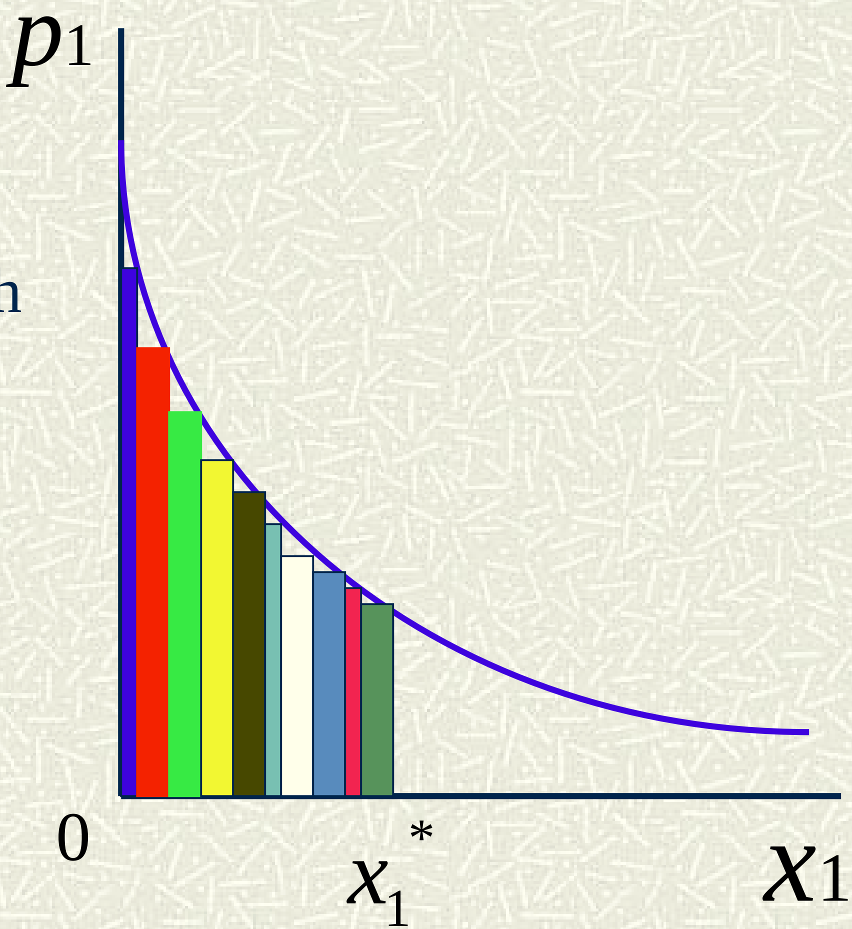
$$p_1 = -MRS$$

Inverse Demand Curve



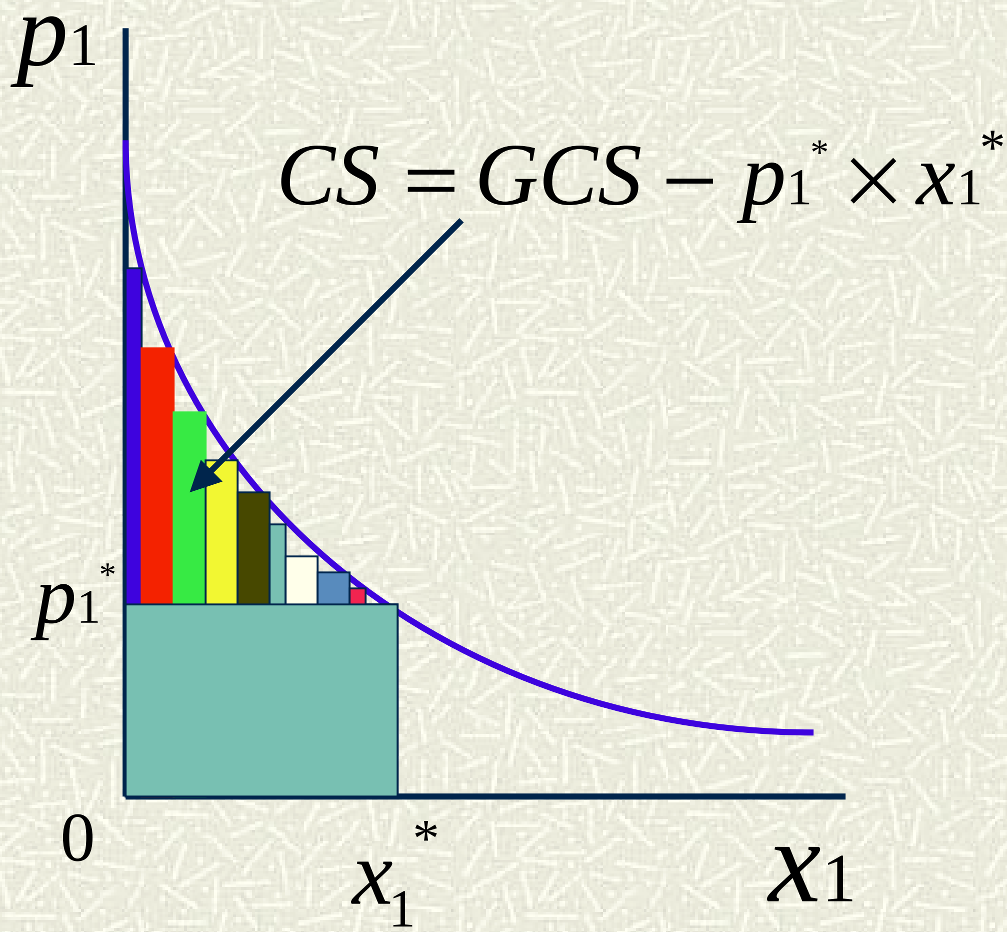
Gross Consumer Surplus

- Consumer buys x^*_1 units of good 1.
- Consumer has different willingness to pay for each extra unit.
- GCS: Area under demand curve.
- GCS tells us how much money consumer willing to pay for x^*_1



Consumer Surplus

- Consumer buys x^*_1 units of good 1.
- Consumer pays p_1^* for each unit.



The Welfare Effect of Changes in Prices

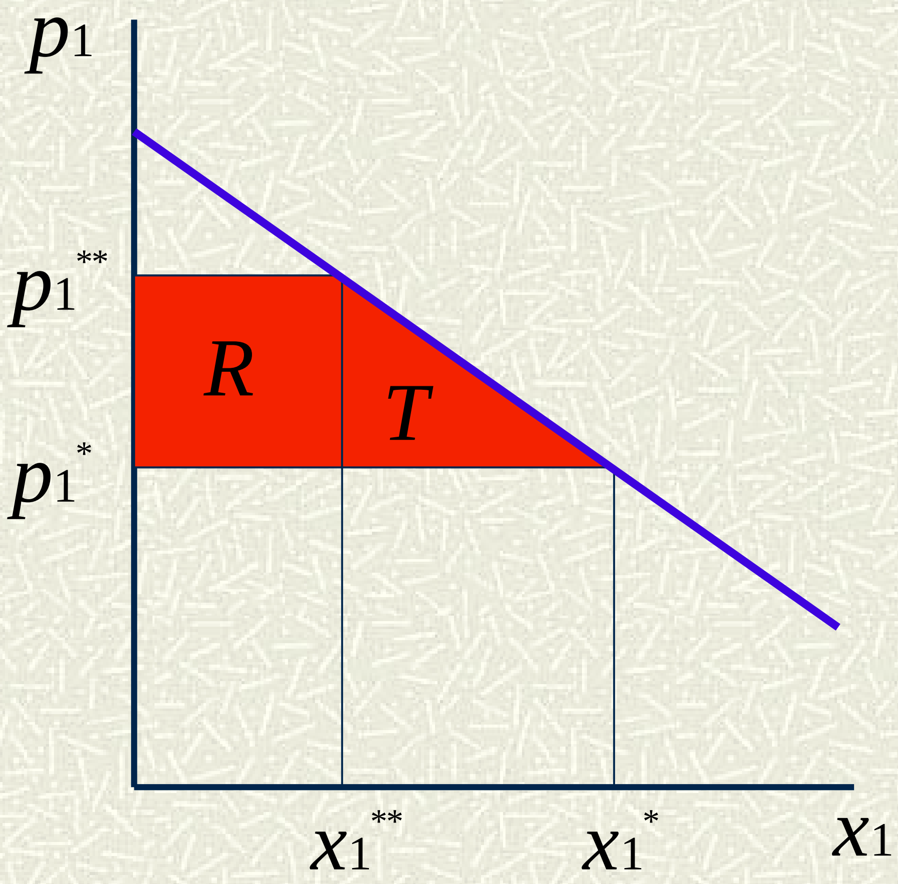
- Goal: provide a monetary measure of the effects of price changes on the utility of the consumer.
- 3 ways of doing it:

1. Compute changes in consumer's surplus;
2. Compensating variation;
3. Equivalent variation.

Change in Consumer's Surplus

- Suppose a tax increases price of good 1 from p_1^* to p_1^{**}.
- Decrease in CS:

$$R + T$$



Change in Consumer's Surplus

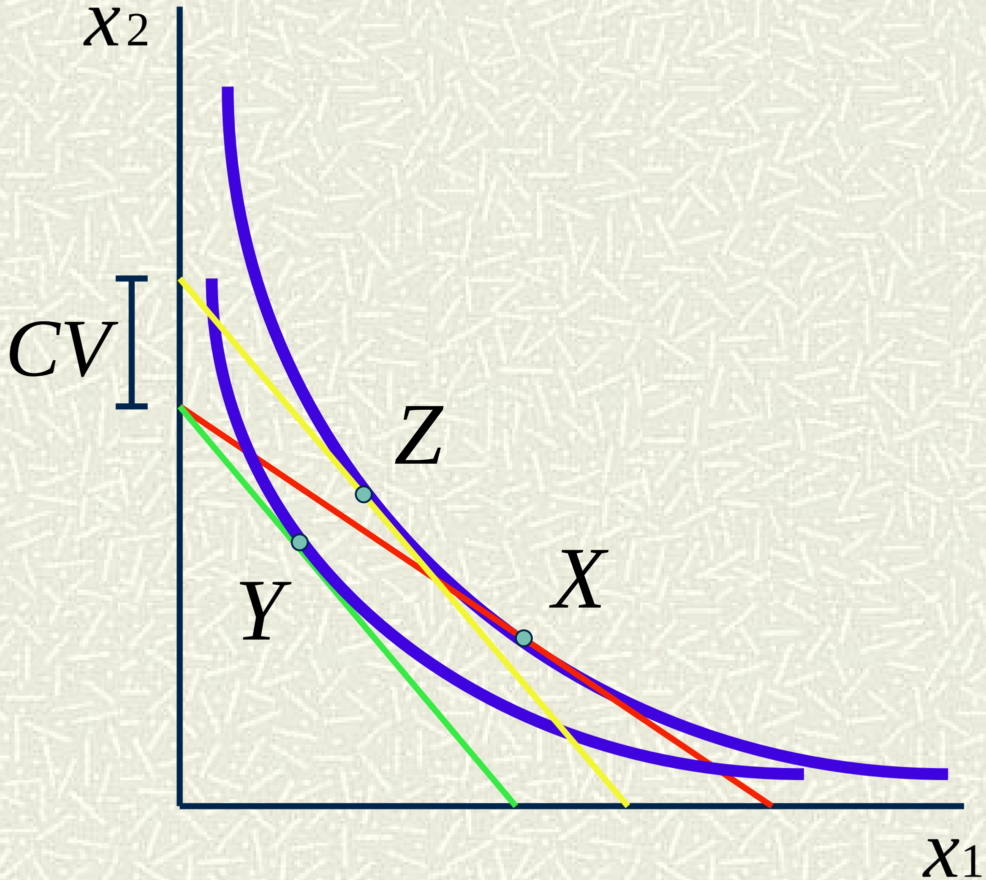
- In practice, to **compute** the change in CS we need to have an estimate of the consumer's demand function. This can be done using statistical methods.
- How is change in CS related to change in utility? The two coincide when utility is quasi-linear:

$$u(x_1, x_2) = v(x_1) + x_2$$

Compensating Variation

- CV=how much money we need to give the consumer **after** the price change to make him just as well off as he was **before** the price change.
- Budget line:

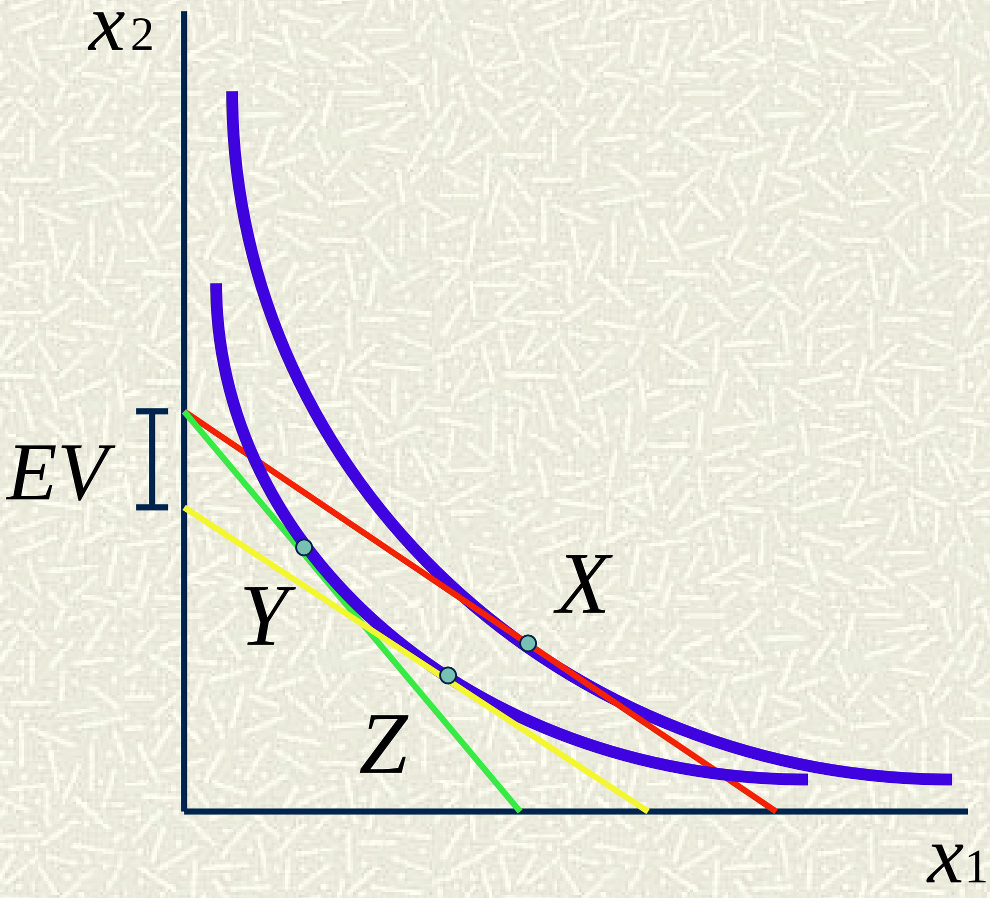
$$x_2 = m - p_1 x_1$$



Equivalent Variation

- EV=how much money we need to take away from the consumer **before** the price change to make him just as well off as he was **after** the price change.
- Budget line:

$$x_2 = m - p_1 x_1$$



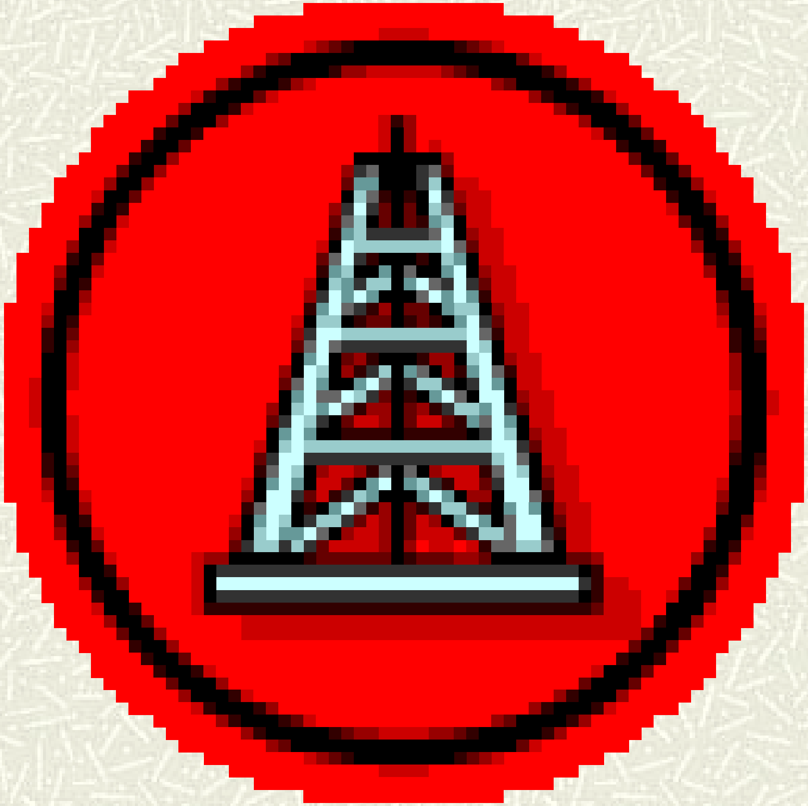
Compensating and Equivalent Variations

- To compute CV and EV we need to know utility function of the consumer.
- This can be estimated from the data by observing consumer’s demand behavior.
- E.g. observe consumer’s choices at different prices and income levels. Observe that expenditures shares are relatively constant:

Cobb-Douglas preferences.

An Example: Increase in Oil Prices

- Often, OPEC manages to restrict production and significantly increase oil prices.
- What’s the effect of this increase on consumers’ welfare?



Model

- Consumers' utility function over gasoline x_1 and composite goods, x_2 :

$$u(x_1, x_2) = 10(x_1)^{\frac{1}{2}} + x_2$$

- Moreover: $m = \$200$

$$p_1^{*} = \$1; p_1^{**} = \$2.$$

Find Consumer Demand's Before Price Increase

- Consumer solves:

$$\max_{x_1, x_2}\left(10 x_1^{\frac{1}{2}} + x_2\right)$$

$$s.t.$$

$$200 = p_1^* x_1 + p_2^* x_2$$

- Optimality condition:

$$-\frac{5}{x_1^{\frac{1}{2}}} = -\frac{p_1^*}{p_2^*} = -p_1^*$$

Find Consumer Demand's Before Price Increase

- Since: $p_1^* = \$1$
- Demand for gasoline is:

$$x_1^* = 25\frac{1}{\left(p_1^*\right)^2} = 25$$

- Demand for composite good:

$$x_2^* = 200 - 25 = 175.$$

Find Consumer Demand's After Price Increase

- Since: $p_1^{**} = \$2$

- Demand for gasoline goes down:

$$x_1^{**} = 25\frac{1}{\left(p_1^{**}\right)^2} = \frac{25}{4}$$

- Demand for composite good:

$$x_2^{**} = 200 - 2\frac{25}{4} = 187.5$$

Compute Change in Consumer's Surplus

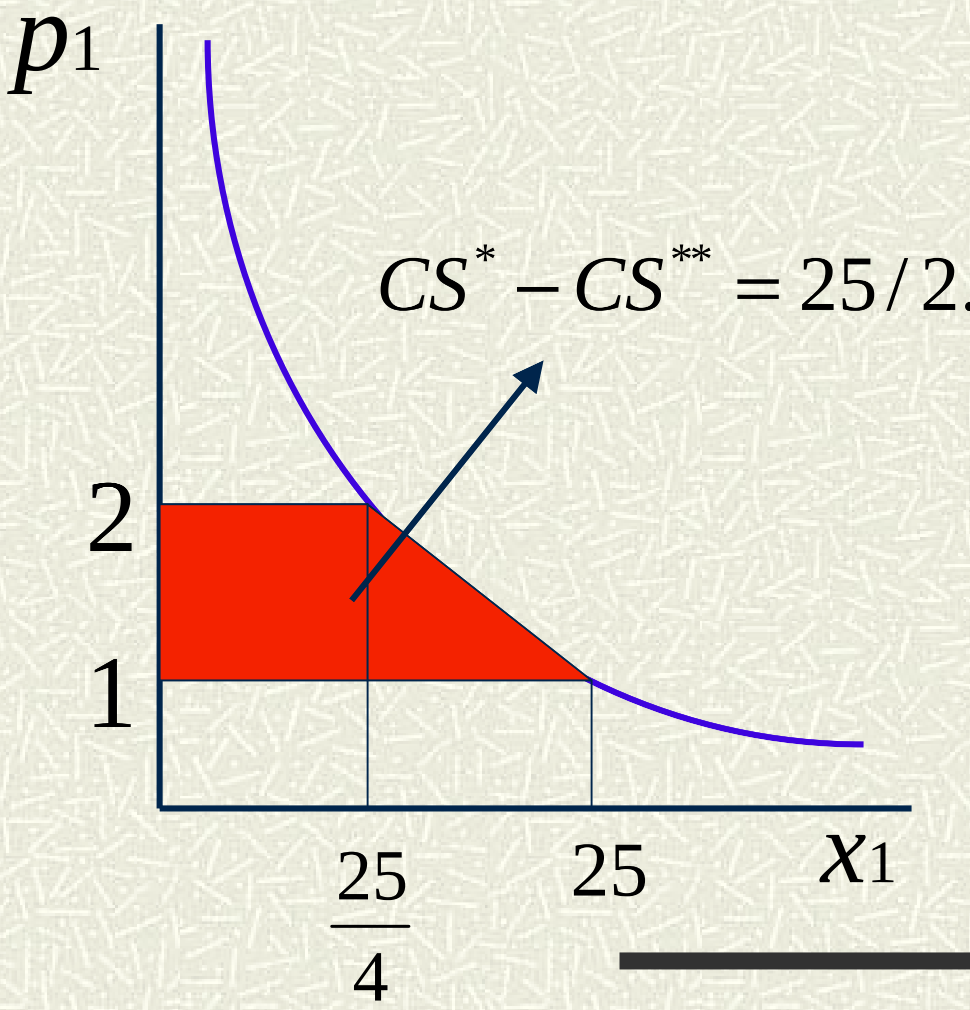
- Inverse demand function:

$$p_1 = \frac{5}{(x_1)^{\frac{1}{2}}}$$

- Consumer surplus:

$$CS^* = 25$$

$$CS^{**} = 25/2.$$



Compute Compensating Variation

- Government pays amount CV such that:

$$u(25,175) = u\left(\frac{25}{4}, 187.5 + CV\right)$$

- Plug in numbers:

$$10(25)^{\frac{1}{2}} + 175 = 10\left(\frac{25}{4}\right)^{\frac{1}{2}} + 187.5 + CV$$

Compute Compensating Variation

- Plug in numbers:

$$10(25)^{\frac{1}{2}} + 175 = 10\left(\frac{25}{4}\right)^{\frac{1}{2}} + 187.5 + EV$$

- Get:

$$EV = \frac{25}{2}$$

Compute Equivalent Variation

- Government pays amount EV such that:

$$u\left(\frac{25}{4}, 187.5\right) = u(25, 175 - EV)$$

- Plug in numbers:

$$10\left(\frac{25}{4}\right)^{\frac{1}{2}} + 187.5 = 10(25)^{\frac{1}{2}} + 175 - EV$$

Compute Equivalent Variation

- Plug in numbers:

$$10\left(\frac{25}{4}\right)^{\frac{1}{2}} + 187.5 = 10(25)^{\frac{1}{2}} + 175 - EV$$

- Get:

$$EV = \frac{25}{2}$$

Conclusion

- In this case: change in consumer’s surplus equals compensating variation which equals equivalent variation.
- In general these three measures differ.

This Wednesday:

Who Wants to be an Economist?