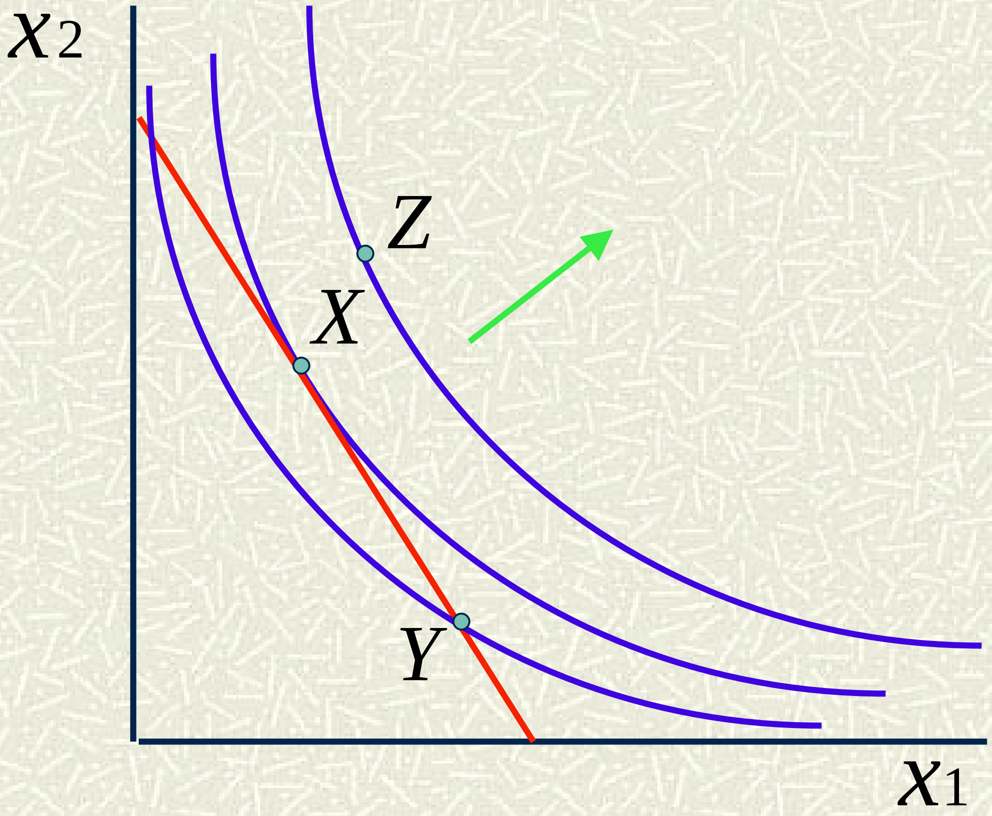
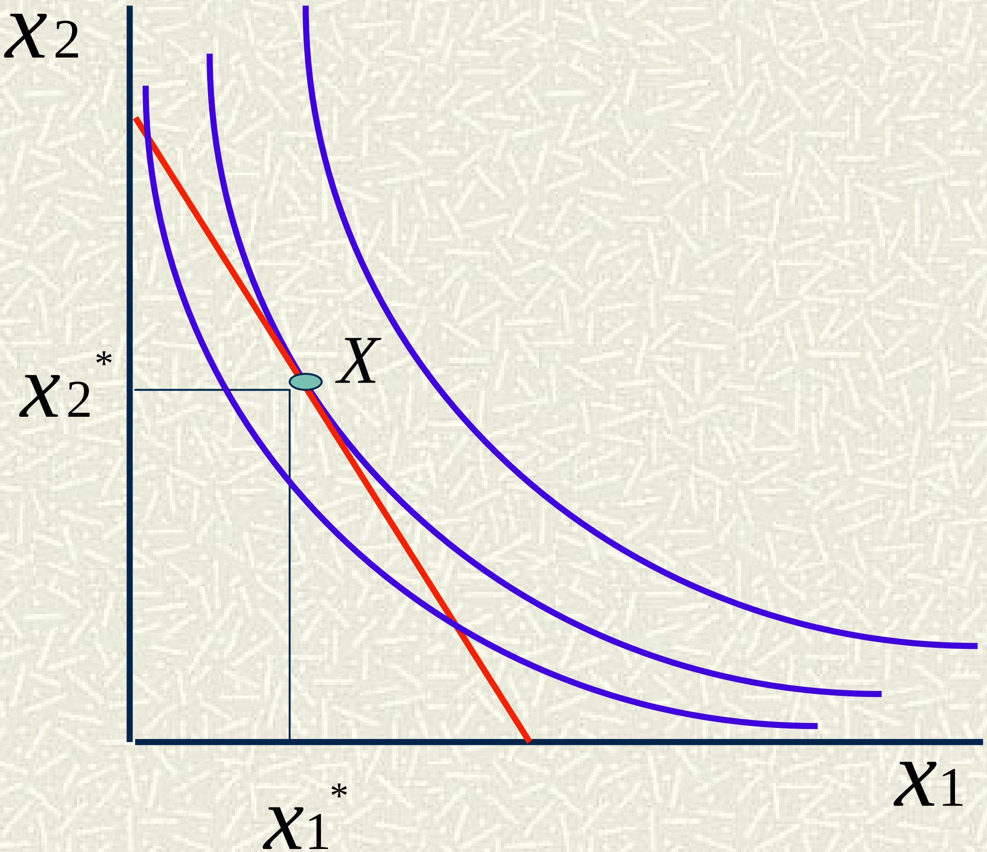
# Choice

# Q: What is the Optimal Choice?



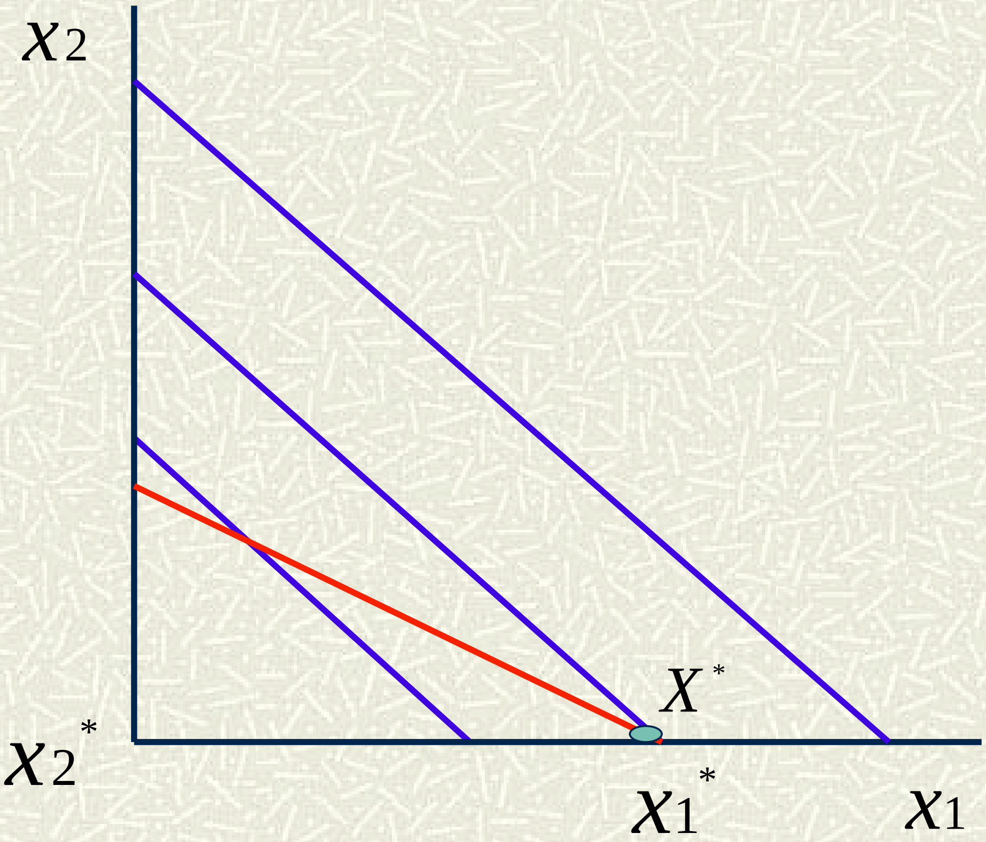

- Budget constraint
- Indifference curves
- More preferred bundles

# A: Optimal Choice is X

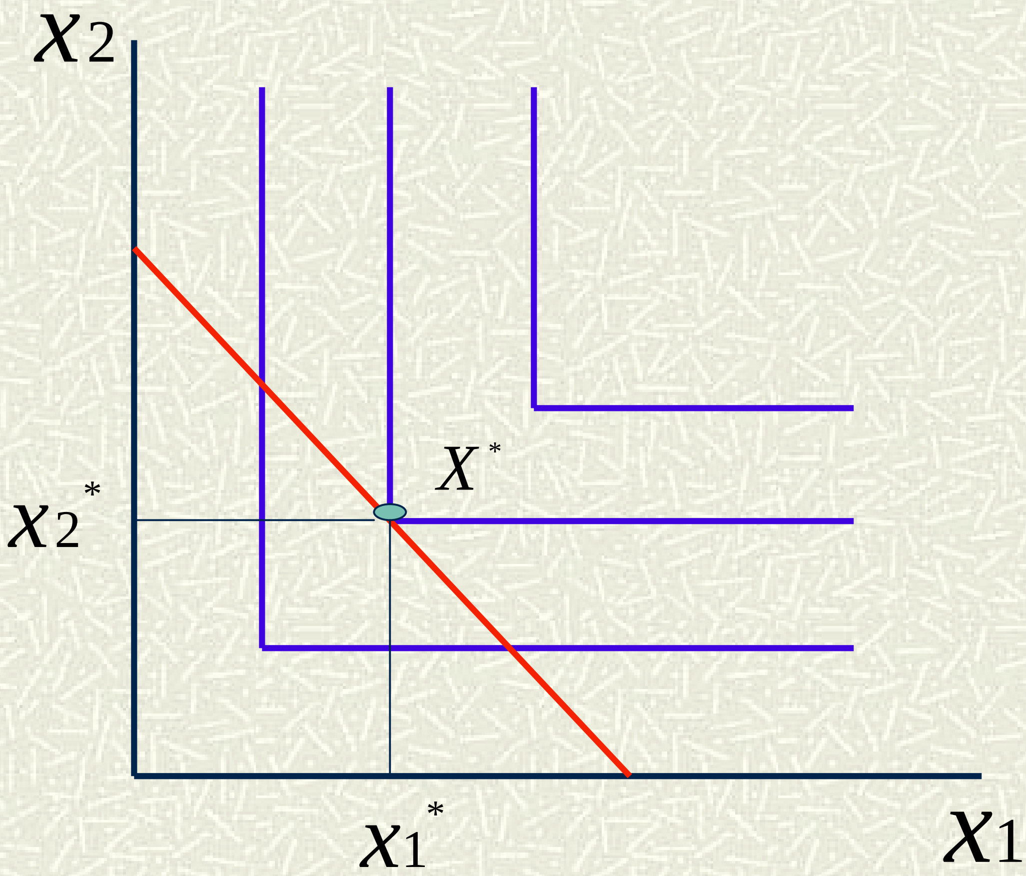



- Optimal choice: indifference curve tangent to budget line.
- Does this tangency condition necessarily have to hold at an optimal choice?

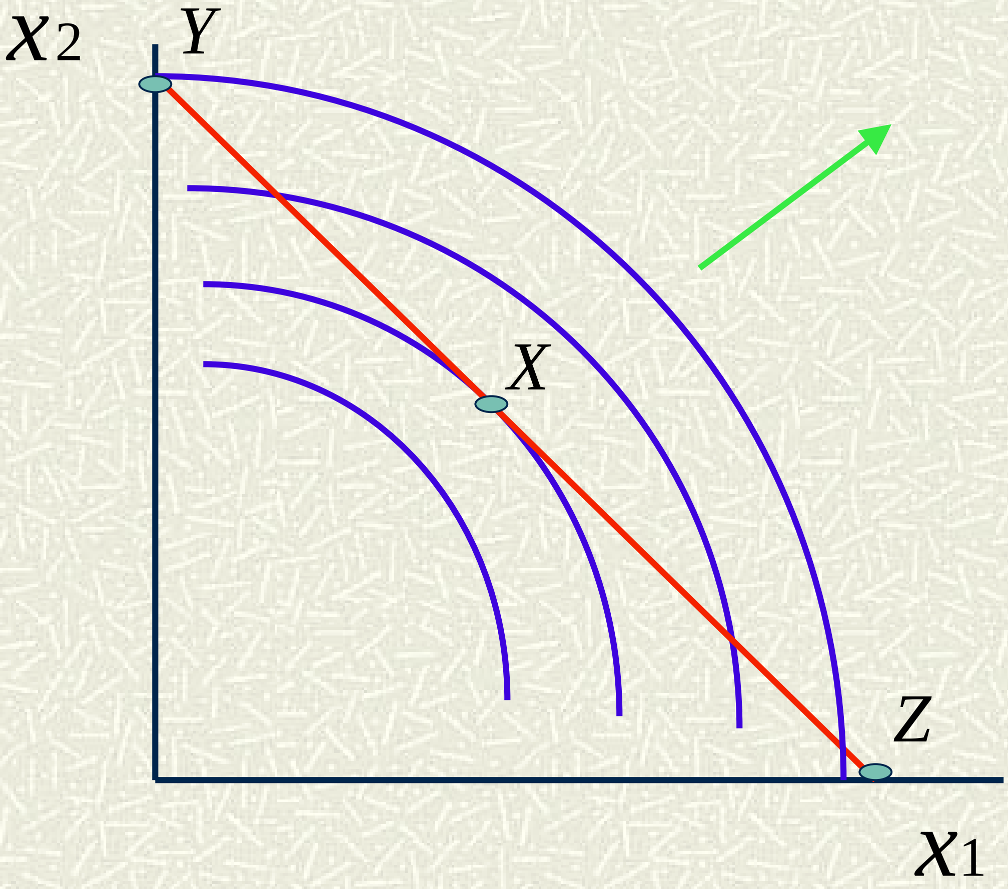
# Perfect Substitutes



# Perfect Complements



# Q: Is Tangency Sufficient?



# What is the General Rule?

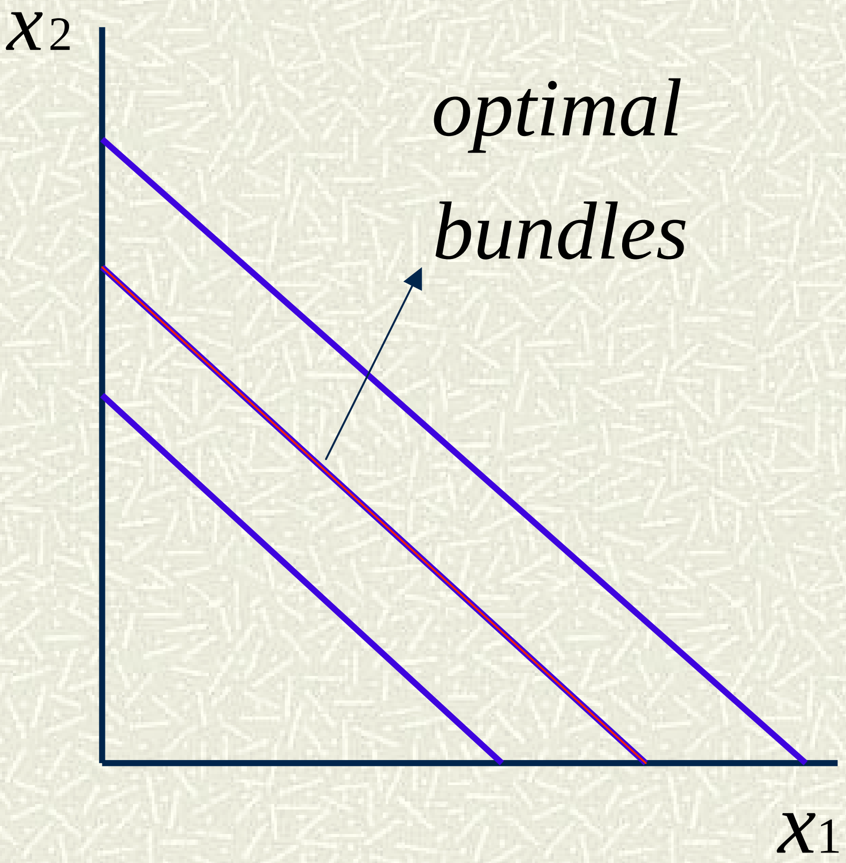
If:

- Preferences are well-behaved.
- Indifference curves are “smooth” (no kinks).
- Optima are interior.

Then:

Tangency between budget constraint and indifference curve is **necessary** and **sufficient** for an optimum.

# Multiple Optima



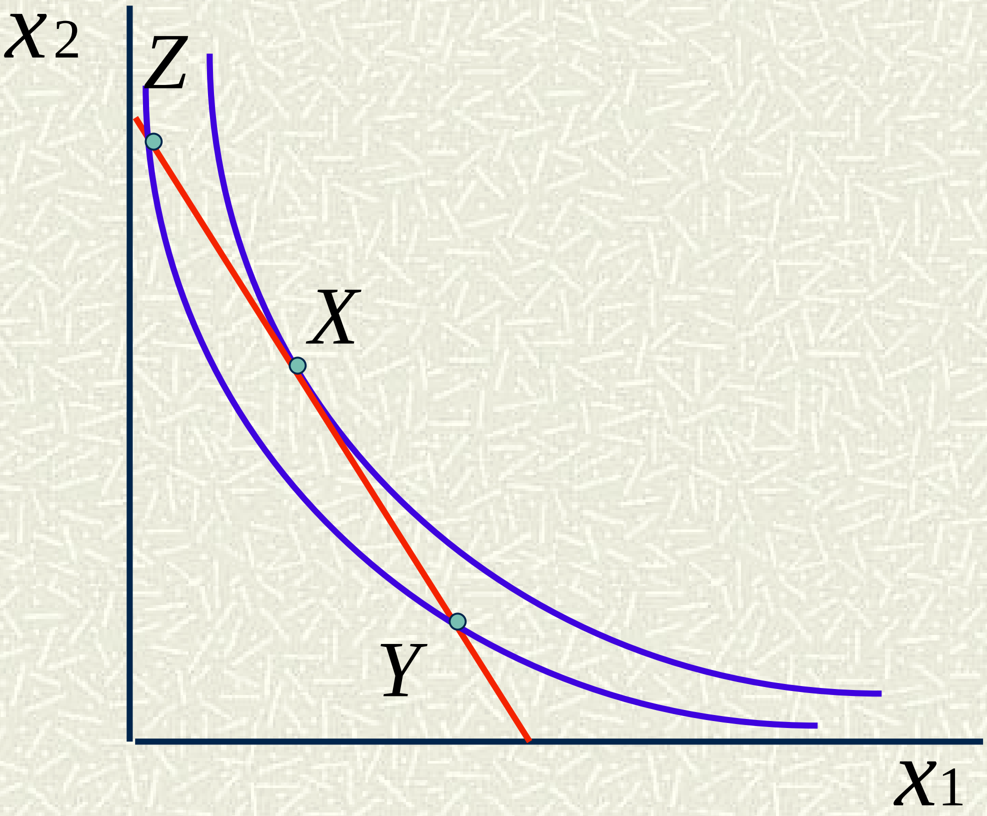

- A way to avoid multiplicity of optima, is to assume **strictly convex** preferences.
- This assumption rules out “flat spots” in indifference curves.

# Economic Interpretation

- At optimum: “Tangency between budget line and indifference curve.”
- Slope of budget line: $$-\frac{p_1}{p_2}$$
- Slope of indifference curve: $MRS(x_1, x_2)$
- Tangency:

$$-\frac{p_1}{p_2} = MRS(x_1, x_2)$$

# Interpretation




- At Z:

$$\frac{p_1}{p_2} < -MRS(x_1, x_2)$$

- At Y:

$$\frac{p_1}{p_2} > -MRS(x_1, x_2)$$

# Tangency with Many Consumers

- Consider many consumers with different preferences and incomes, facing the same prices for goods 1 and 2.

- Q: Why is it the case that at their optimal choice $\left(x_{i1}^{*}, x_{i2}^{*}\right)$ the MRS between 1 and 2 for different consumers is equalized?

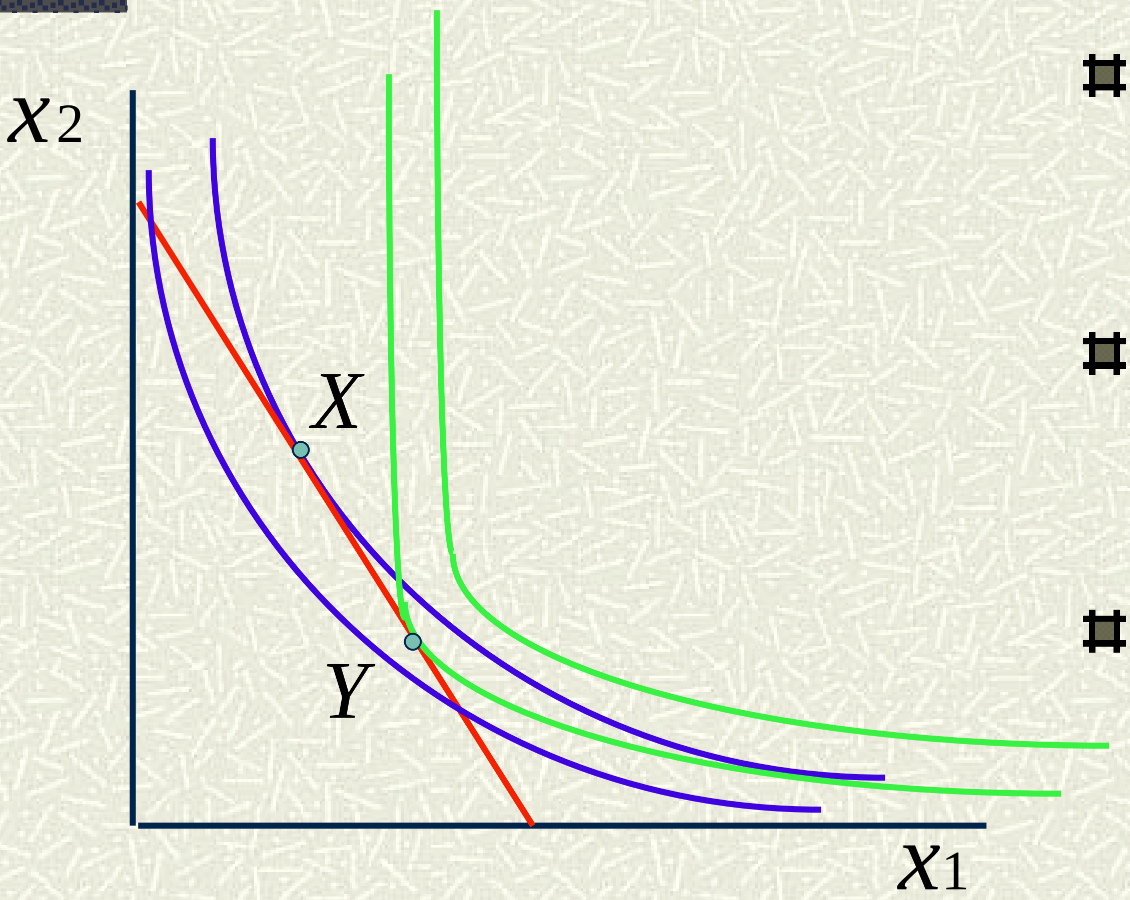
# Tangency with Many Consumers

- A: Because if a consumer $i$ makes an optimal choice, then:

$$-\frac{p_1}{p_2} = MRS_i(x_{i1}^*, x_{i2}^*)$$

- Implication: everyone who is consuming the two goods must agree on how much one is worth in terms of the other.

# Tangency with 2 Consumers




- Indifference curves of consumer 1:
- Indifference curves of consumer 2:
- Budget line:

# Finding the Optimum in Practice: a Cobb-Douglas Example

- Preferences represented by:

$$u(x_1, x_2) = c \log x_1 + (1-c) \log x_2$$

- Budget line:

$$p_1 x_1 + p_2 x_2 = m$$

# Finding the Optimum in Practice: a Cobb-Douglas Example

Mathematically, we would like to:

$$\max_{x1,x2}\left[c\log x_1 + (1-c)\log x_2\right]$$

such that

$$p_1x_1 + p_2x_2 = m$$

# Finding the Optimum in Practice: a Cobb-Douglas Example

- Replace budget constraint into objective function:

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

- New problem:

$$\max_{x1} \left[ c \log x_1 + (1-c) \log\left( \frac{m}{p_2} - \frac{p_1}{p_2} x_1 \right) \right]$$

# Finding the Optimum in Practice: a Cobb-Douglas Example

- New problem:

$$\max_{x_1}\left[c\log x_1+(1-c)\log\left(\frac{m}{p_2}-\frac{p_1}{p_2}x_1\right)\right]$$

- First-Order Condition:

$$\frac{c}{x_1}-(1-c)\left(\frac{p_2}{m-p_1x_1}\right)\frac{p_1}{p_2}=0$$

# Finding the Optimum in Practice: a Cobb-Douglas Example

- First-Order Condition:

$$\frac{c}{x_1} - (1-c)\left(\frac{p_2}{m - p_1 x_1}\right)\frac{p_1}{p_2} = 0$$

- Rearranging:

$$-\frac{p_1}{p_2} = -\frac{c}{(1-c)}\left(\frac{m - p_1 x_1}{p_2 x_1}\right) = MRS(x_1, x_2)$$

# Finding the Optimum in Practice: a Cobb-Douglas Example

- First-Order Condition:

$$-\frac{p_1}{p_2} = -\frac{c}{(1-c)}\left(\frac{m - p_1 x_1}{p_2 x_1}\right)$$

- Solve for $x_1$: $x_1^* = c\frac{m}{p_1}$

- Expenditures share in 1: $\frac{p_1 x_1^*}{m} = c$

# Finding the Optimum in Practice: a Cobb-Douglas Example

- Q: How do I find $x_2^*$ ?

- A: Use the budget constraint:

$$x_2^* = \frac{m}{p_2} - \frac{p_1}{p_2} x_1^* = \frac{m}{p_2} - \frac{p_1}{p_2} c \frac{m}{p_1}$$

$$= (1 - c)\frac{m}{p_2}$$