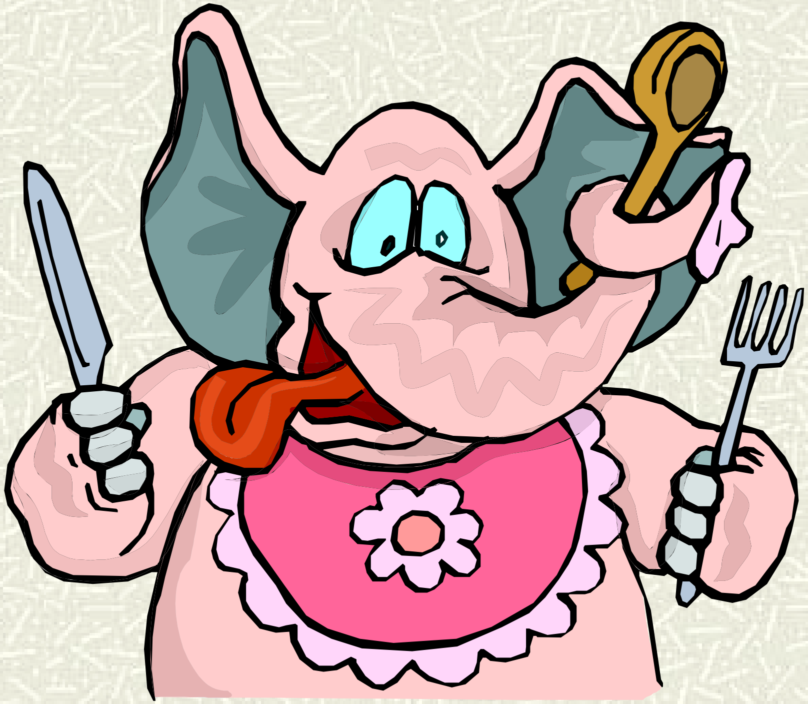
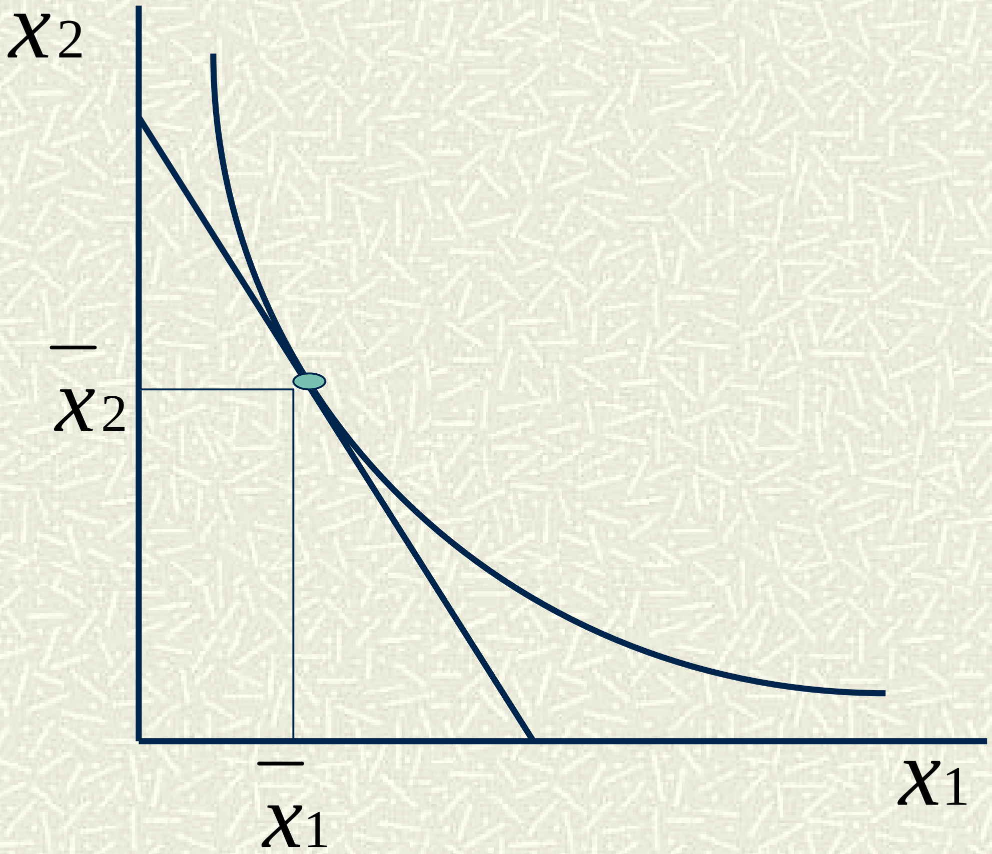
Utility



Marginal Rate of Substitution

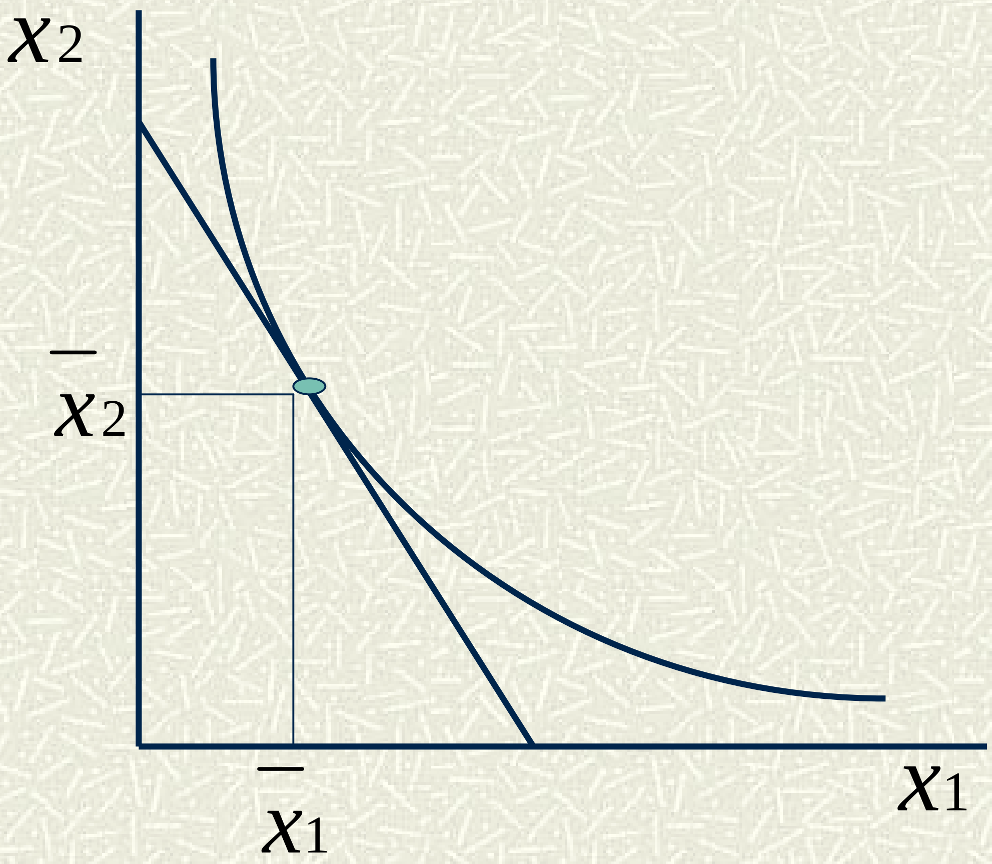


- The MRS is the slope of the indifference curve at a point $(\bar{x}_1, \bar{x}_2)$
- MRS=derivative of indifference curve

Interpretation of MRS

- The MRS measures the rate at which the consumer is willing (i.e., indifferent) to substitute one good for the other.
- If good 2 is measured in dollars, the MRS measures the consumer's willingness to pay for an extra unit of good 1.

Assumptions on Preferences and the MRS



- Monotonicity: MRS negative
- (Strict) Convexity: MRS decreases as x_1 increases

Utility Function

- Idea: assign a number to each consumption bundle, with higher numbers assigned to more-preferred bundles.
- A utility function $u(x_1, x_2)$ represents a preference relation $\succ$:

$$(x_1, x_2) \succ (y_1, y_2) \longleftrightarrow u(x_1, x_2) > u(y_1, y_2)$$

Utility Function: Does It Always Exist?

- Q: Given a preference relation $\succ$ can we find a utility function that represents it?
- A: If preferences are complete and transitive (plus a technical assumption called "continuity" is verified) we can. [Sufficient condition]

Is Transitivity Necessary?

- Q: Is it necessary that preferences are transitive for the existence of a utility function that represents them?
- A: Yes. Otherwise:

$$(1,1) \succ (1,0.5) \succ (0.4,1) \succ (1,1)$$

$$u(1,1) > u(1,0.5) > u(0.4,1) > u(1,1)$$

Utility is Just Ordinal

E.g.: (1,1) $\succ$ (1,0.5) $\succ$ (0.4,1) $\sim$ (0.8,0.8)

Can be represented in different ways:

Bundle	$u(x_1, x_2)$	$2u(x_1, x_2)$	$u(x_1, x_2) - 4$
(1,1)	3	6	-1
(1,0.5)	2	4	-2
(0.4,1)	1	2	-3
(0.8,0.8)	1	2	-3

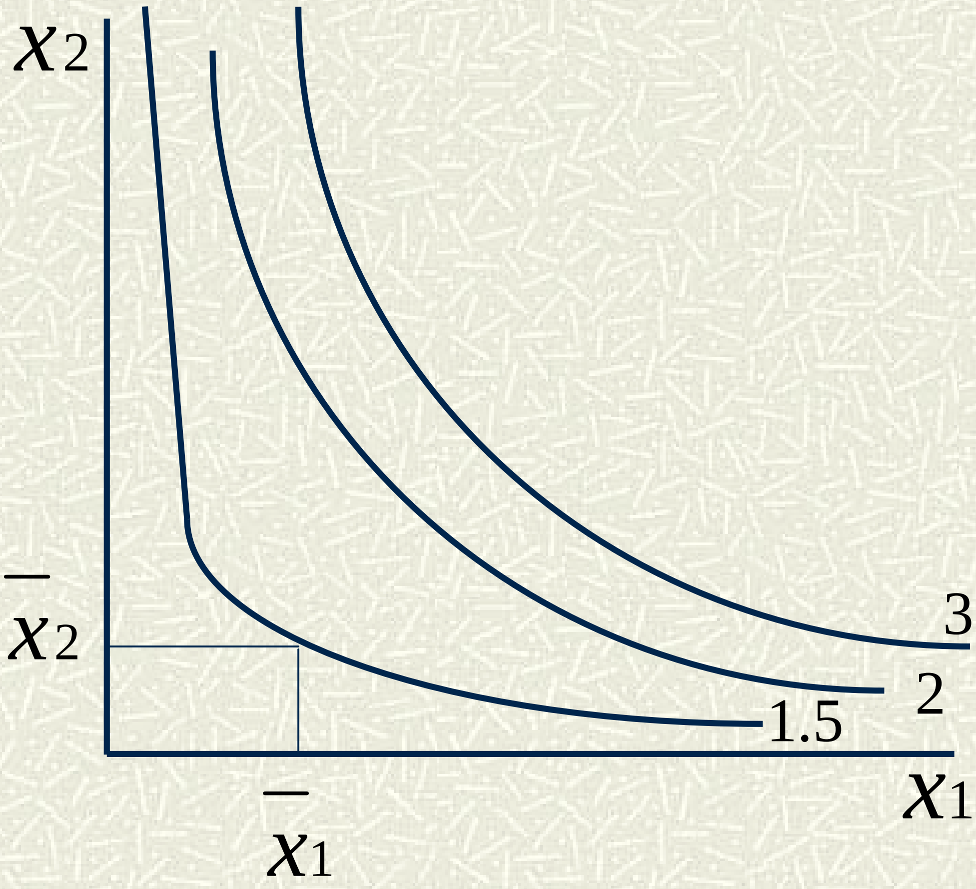
In General

If :

- 1) $u(x_1, x_2)$ represents $\succ$
- 2) $f(u)$ is a positive monotonic transformation $[u_1 > u_2 \longrightarrow f(u_1) > f(u_2)]$

Then, also $f(u(x_1, x_2))$ represents $\succ$

Utility Functions and Indifference Curves



- Utility function:

$$u(x_1, x_2) = x_1 x_2$$

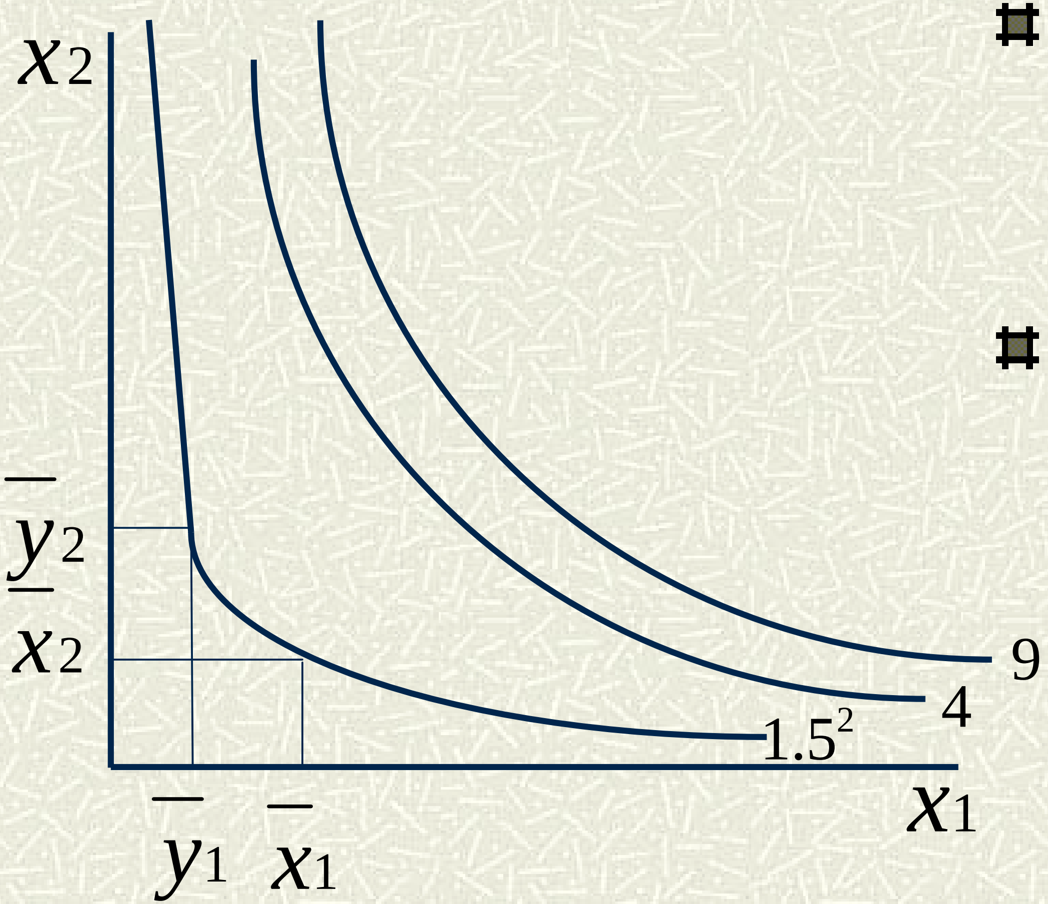
- Indifference curves:

$$x_1 x_2 = \bar{u}$$

or:

$$x_2 = \frac{\bar{u}}{x_1}$$

Indifference Curves and Monotonic Transformations



- Utility function:

$$v(x_1, x_2) = x_1^2 x_2^2$$

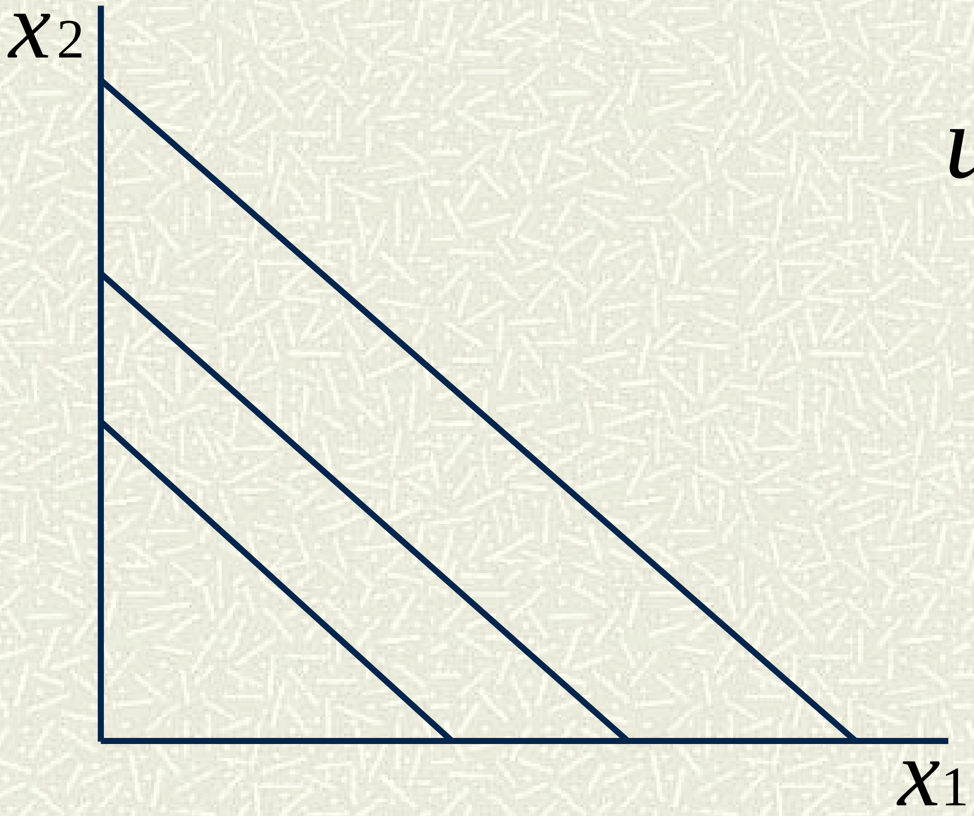
- Indifference curves:

$$x_1^2 x_2^2 = \bar{v}$$

or:

$$x_2 = \frac{\sqrt{\bar{v}}}{x_1}$$

Perfect Substitutes



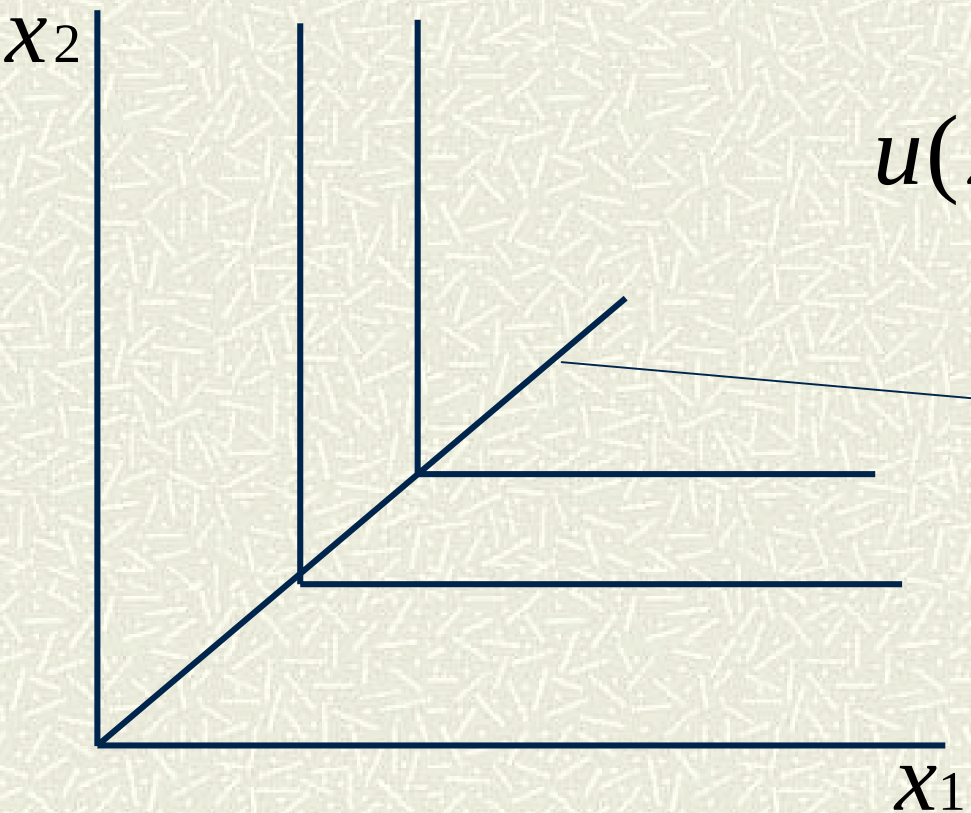
- Utility function:

$$u(x_1, x_2) = ax_1 + bx_2$$

- Indifference curves:

$$x_2 = \frac{\bar{u}}{b} - \frac{a}{b}x_1$$

Perfect Complements

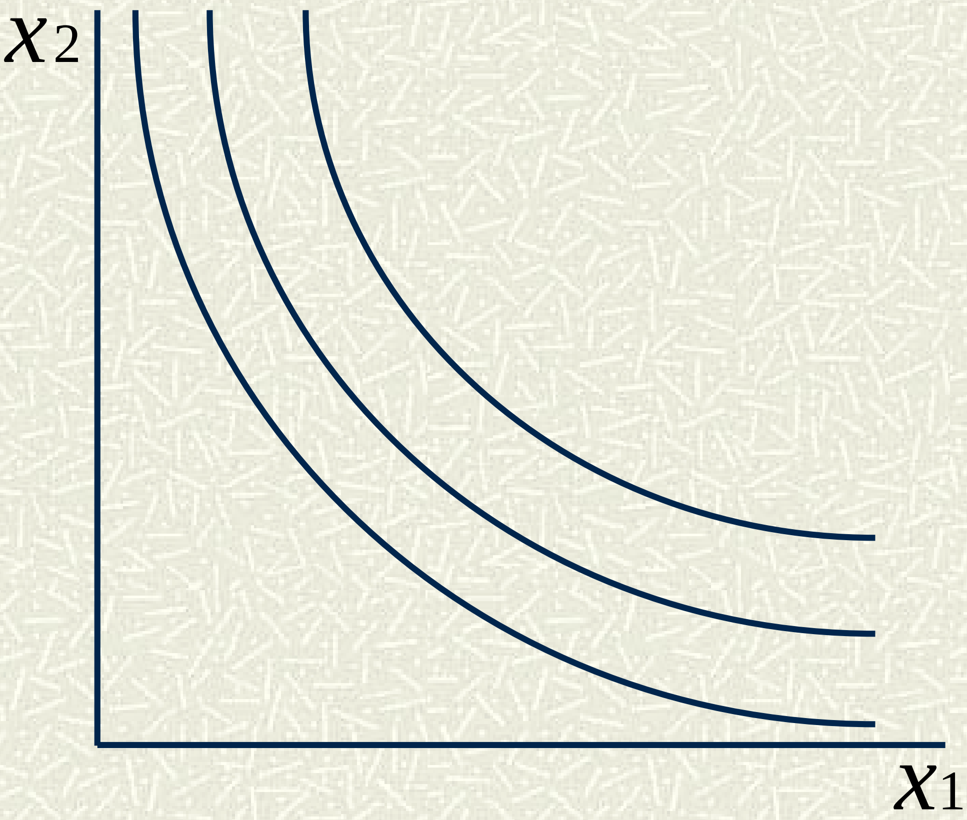


- Utility function:

$$u(x_1, x_2) = \min(ax_1, bx_2)$$

$$x_2 = \frac{a}{b} x_1$$

Cobb-Douglas Preferences



- Utility function:

$$u(x_1, x_2) = x_1^{\,c}\, x_2^{\,d}$$

- Indifference curves:

$$x_2 = \frac{\bar{u}^{\frac{1}{d}}}{x_1^{\frac{c}{d}}}$$

Cobb-Douglas

- Most commonly used utility function in economics
- Possible to assume without loss of generality that $c + d = 1$
- Why?: apply $f(u) = u^{\frac{1}{c+d}}$ to get

$$v(x_1, x_2) = x_1^{\frac{c}{c+d}} x_2^{\frac{d}{c+d}}$$

Marginal Utility

- Consider a consumer that is consuming

$$(x_1, x_2)$$

- Q: By how much does his utility change as we increase by a very small amount his consumption of good 1?
- A: Marginal utility of good 1: $$\frac{\partial u(x_1, x_2)}{\partial x_1}$$

Marginal Utility and Units

- The marginal utilities of good 1 and good 2 depend on the specific utility function we are using.
- Consider: $v(x_1, x_2) = f(u(x_1, x_2))$
- Then: $\dfrac{\partial v(x_1, x_2)}{\partial x_1} = \dfrac{\partial f(u(x_1, x_2))}{\partial u} \dfrac{\partial u(x_1, x_2)}{\partial x_1}$

Marginal Utility and MRS

- MRS only depends on preferences and not on their specific representation (utility function).
- Marginal utilities can be used to compute the MRS between two goods.

Computing the MRS

- Consider an indifference curve: $u(x_1, x_2) = \bar{u}$
- Q: What is the slope of this indifference curve (MRS)?
- A:

$$MRS(x_1, x_2) = \frac{\partial x_2}{\partial x_1} = -\frac{\dfrac{\partial u(x_1, x_2)}{\partial x_1}}{\dfrac{\partial u(x_1, x_2)}{\partial x_2}}$$

The MRS and Monotonic Transformation

- Consider a monotonic transformation

$$v(x_1, x_2) = 2u(x_1, x_2)$$

- Q: What is the MRS in this case?
- A:

$$MRS(x_1, x_2) = -\frac{2\dfrac{\partial u(x_1, x_2)}{\partial x_1}}{2\dfrac{\partial u(x_1, x_2)}{\partial x_2}} = -\frac{\dfrac{\partial u(x_1, x_2)}{\partial x_1}}{\dfrac{\partial u(x_1, x_2)}{\partial x_2}}$$