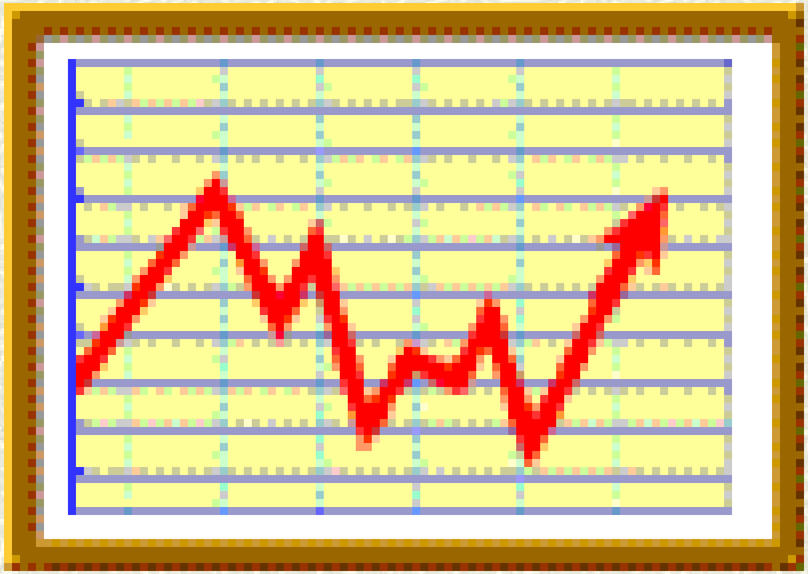
Profit Maximization (Cont'd)



- Renting or buying capital
- Profit maximization and returns to scale

Renting Capital

- If physical capital is one of the firm's inputs, the firm can either **rent** capital or **buy** it
- E.g.: firm can lease computers
- Problem solved by firm:

$$\max\left[pF(K,L) - w_L L - w_K K\right]$$

Buying Capital

- What problem would the firm solve in the case it decides to **buy** rather than to **rent** capital?
- Buying a machine has an impact on the firm's revenue for several years
- **Q:** how do we compare revenue tomorrow to revenue today? How do we account for risk?

No Uncertainty

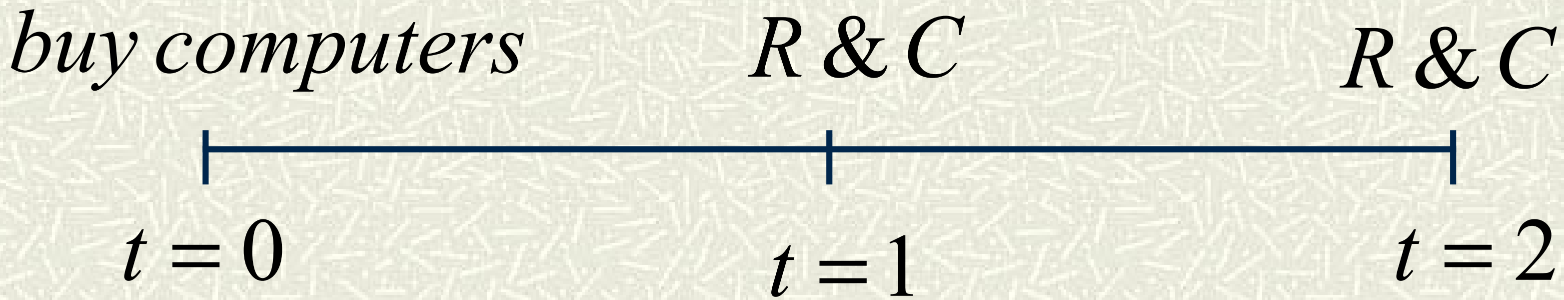
Firm can borrow and lend at interest rate $r=0.10$

Firm is considering how many computers to buy today. Each computer:

- Costs $10,000
- Will be used for two years and then discarded (zero resale value)

No Uncertainty

Objective of the firm is to maximize the **present value of profits**: **present value** of **revenues** minus the **present value** of **costs**



Computing the Present Value

- What is the value today of having \$1 one year from now, if the interest rate is r=0.1?

$$\frac{\$1}{1+0.1} \approx \$0.9$$

- What is the value today of having \$1 two years from now?

$$\frac{\$1}{(1+0.1)^2} \approx \$0.82$$

Present Value of the Firm at t=0

$$PV = -p_k K + \frac{R_1 - C_1}{(1+0.1)} + \frac{R_2 - C_2}{(1+0.1)^2}$$

$$R_1 - C_1 = p_1 F(K, L_1) - w_{L1} L_1$$

$$R_2 - C_2 = p_2 F(K, L_2) - w_{L2} L_2$$

Maximizing the Present Value

- The firm should decide how many computers to buy and how much labor to hire in order to maximize its **present value**

$$V^* = \max_{K, L_1, L_2} [PV]$$

- **Q:** how much would you be willing to pay to buy this firm at time t=0?

$$V^*$$

What are the Firms Profits?

- Cost of buying computers must be **amortized** across their lifetime
- To construct cost as a **flow** consider:

1. Annual **economic depreciation**

2. Opportunity cost due to foregone interest

User Cost of Capital

- **Year 1:**

1. Annual economic depreciation: $5,000
2. Opportunity cost of funds: ($10,000)0.10

- **Year 2:**

1. Annual economic depreciation: $5,000
2. Opportunity cost of funds: ($5,000)0.10

Profits

Profits=annual revenue-labor cost-user cost of capital

Buy or Rent?

- If the rental rate is **larger** than the user cost, then it is convenient to buy capital
- If the rental rate is **lower** than the user cost, then it is convenient to rent capital
- If the capital market is **competitive**, the rental rate should **equal** the user cost: firm indifferent between buying and renting

Uncertainty

- Suppose there is uncertainty about price of the product firm is selling.
- Problem gets more complicated because:

1. Firm must take expectation of output price
2. Discount factor must be adjusted to take risk into account

Profit Maximization and Returns to Scale

Q: How much profit does a competitive firm with a constant returns to scale technology make in the long-run?

Profit Maximization and Returns to Scale

- **A:** Zero!
- Suppose it makes positive profits:

$$\Pi^* = py^* - w_1 x_1^* - w_2 x_2^* > 0$$

- Double all inputs:

$$2\Pi^* = p\left(2y^*\right) - w_1\left(2x_1^*\right) - w_2\left(2x_2^*\right) > \Pi^*$$

Profit Maximization and Returns to Scale

- Double all inputs:

$$2\Pi^* = p\left(2y^*\right) - w_1\left(2x_1^*\right) - w_2\left(2x_2^*\right) > \Pi^*$$

- This means that the firm was not choosing inputs optimally before! Contradiction!
- Thus, zero profits is the only possibility

Interpretation

Suppose you are the owner of a firm that produces software with a constant returns to scale technology:

$$y = f(x_L, x_M)$$

where x_L represents workers and x_M managers (including yourself)

Interpretation

Then, this firm's profits in the long run are zero:

- Pay wage to workers x_L
- Pay salary to managers x_M (including yourself because of **opportunity cost**)

Nothing else is left