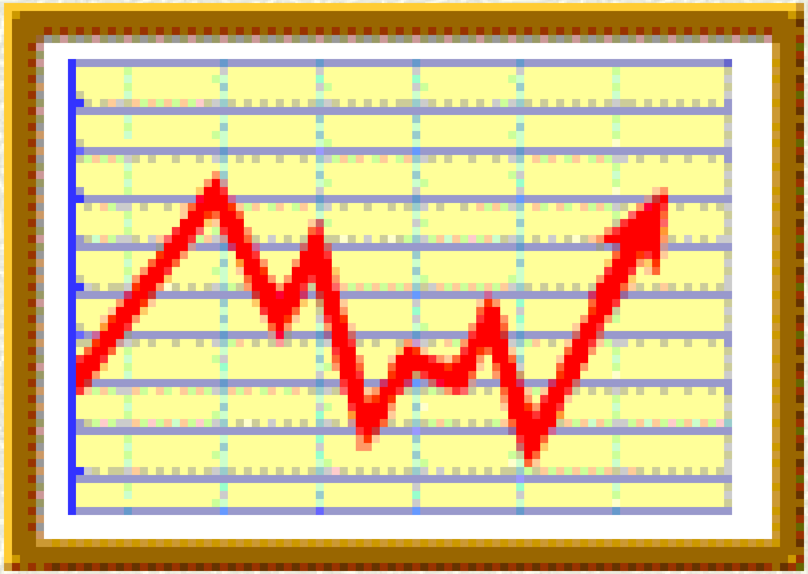
Profit Maximization



- Profits
- The objectives of the firm
- Fixed and variable factors
- Profit maximization in the short and in the long run
- Returns to scale and profits

Competitive Markets

- In a competitive market firms take input and output prices as given
- Firms are “small” relatively to the market, so that their decisions do not affect market prices
- **Q**: When is this assumption reasonable?

Product Homogeneity

- **A**: In markets where firms produce (almost) identical products
- When products are **homogeneous** (agricultural products, raw materials, etc.) no firm can raise its price without losing most of its customers
- Differentiated products: a firm can set its price without losing all customers

The Objective of the Firm

Q: What is the objective of the firm?

A: The maximization of profits

Q: How are profits defined?

Defining Economic Profits

- Profits are defined as **revenues** minus **costs**
- E.g. a firm produces light bulbs using labor and capital

$$\Pi = p_b Y - w_L L - w_K K$$

Defining Economic Profits

- Labor: suppose that the owner of the firm works in the firm
- **Q:** Should costs take the owner's labor into account?
- **A:** Yes, we need to take into account the **opportunity cost** for the owner of not employing his/her labor somewhere else

Defining Economic Profits

- **Q:** How should you take into account the cost of using **physical capital**?

- **A:** Since factor inputs are measured in flows (e.g. labor and machine hours per week), the unit cost of using physical capital should be its **rental rate**

Fixed and Variable Factors

- **Short run**: there are **fixed** factors that the firm is obliged to employ.
- E.g.: firm has signed a contract to lease space in a building for certain number of months. Even if the firm decides not to produce it still has to pay lease.

Fixed and Variable Factors

- **Long run**: all factors of production are **variable**.
- **Variable factors** are factors that the firm employs and pays for only if it decides to produce something. E.g.: labor
- In the long run all factors are variable: firm can always decide to use zero inputs and produce zero output, i.e., go out of business.

Short Run Profit Maximization

- Let’s consider the problem of a consulting firm that has rented 3000sq feet in a building (fixed factor)
- It has to decide how many consultants x_l to hire
- Production function (output in a year):

$$y = (3000)^{0.2}(x_l)^{0.6}$$

Short Run Profit Maximization

- Suppose the salary of a consultant is \$70K per year, office space costs \$1000 per square foot per year, and the firm sells its consultation services at \$100K each
- Problem of the firm in the short run:

$$\max\left((100)(3000)^{0.2}(x_l)^{0.6} - 70x_l - 3000\right)$$

Short Run Profit Maximization

- Problem of the firm in the short run:

$$\max\left(100(3000)^{0.2}(x_l)^{0.6} - 70x_l - 3000\right)$$

- First order condition:

$$(100)0.6(3000)^{0.2}(x_l)^{0.6-1} - 70 = 0$$

Short Run Profit Maximization

- First order condition:

$$(100)0.6(3000)^{0.2}(x_l)^{0.6-1} = 70$$

- Interpretation:

$$p(MP_L) = salary$$

Short Run Profit Maximization

- First order condition:

$$0.6(100)(3000)^{0.2}(x_l)^{0.6-1} = 70$$

- Solve this equation for number of consultants. Then get output and profits :

$$x_l \approx 37 \quad y \approx 43 \quad \Pi \approx -1290K$$

Isoprofit Lines

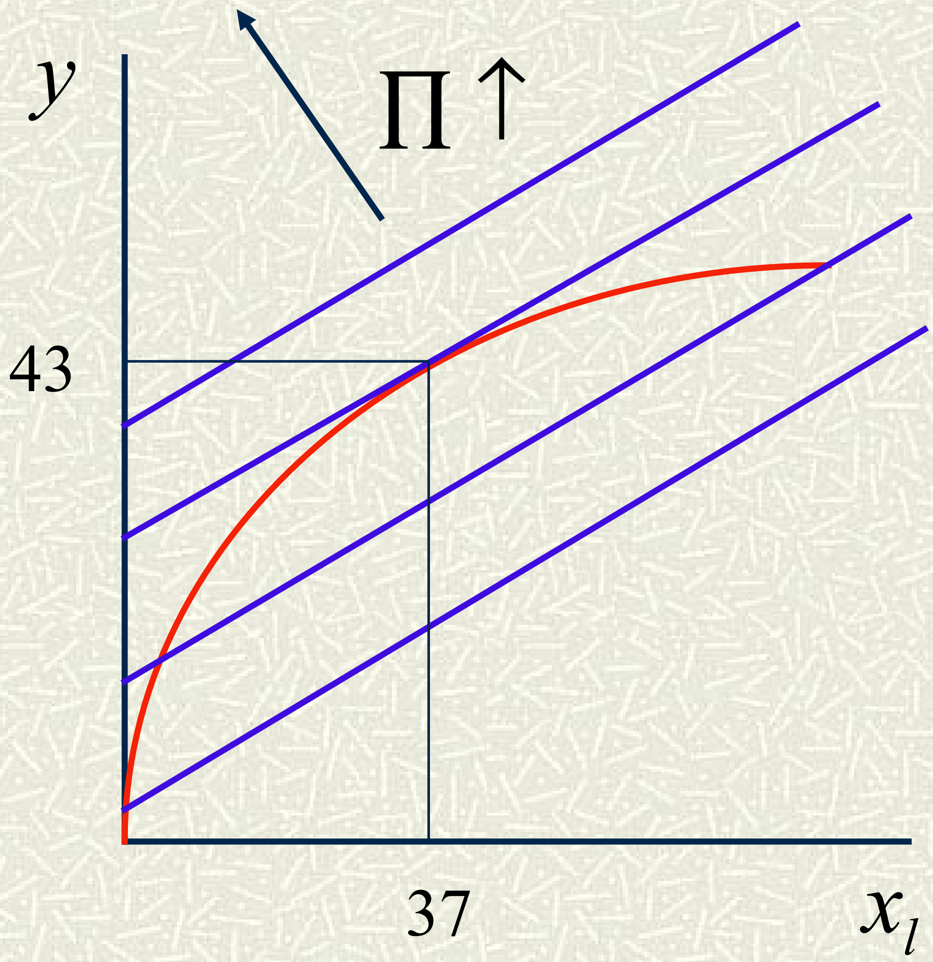
- The firm's profits are given by:

$$\Pi = 100y - 70x_l - 3000$$

- Isoprofit line:

$$y = \frac{\Pi}{100} + \frac{7}{10}x_l + 30$$

Isoprofit Lines



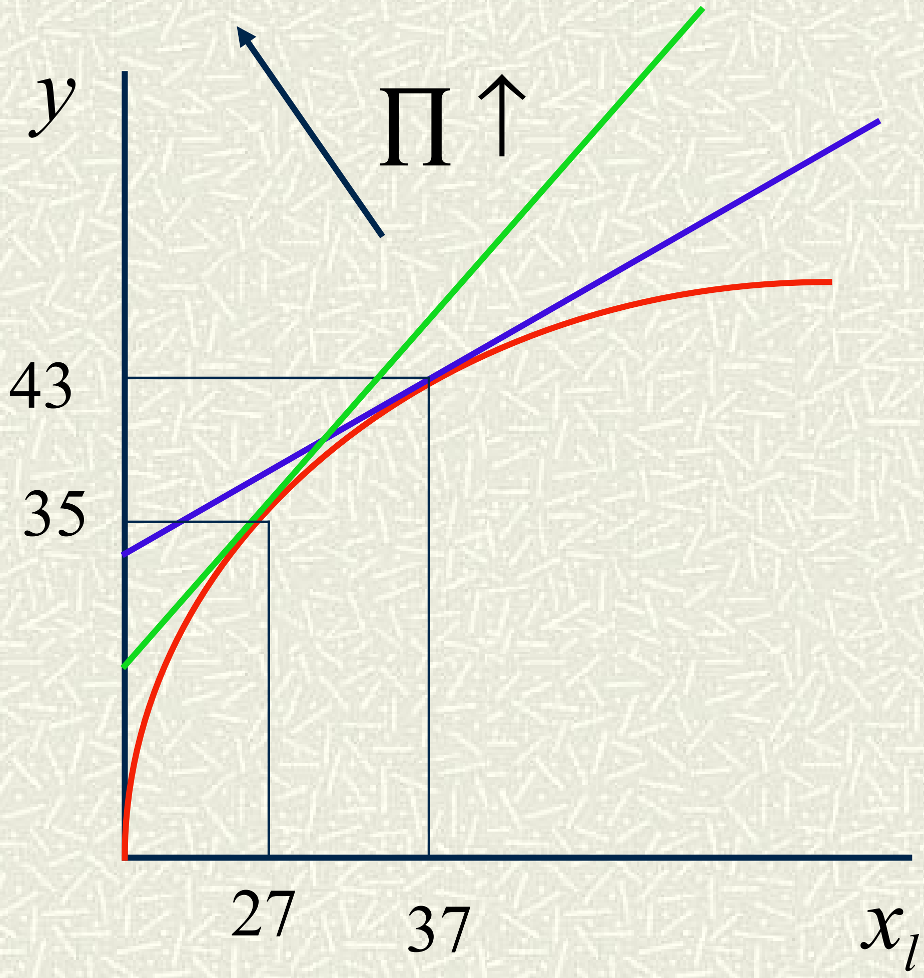
- Isoprofit lines:

$$y = \frac{\Pi}{100} + \frac{7}{10} x_l + 30$$

- Production function:

$$y = (3000)^{0.2} (x_l)^{0.6}$$

Change in Salary: 70K to 80K



- Isoprofit lines:

$$y = \frac{\Pi}{100} + \frac{8}{10} x_l + 5$$

- Production function:

$$y = (3000)^{0.2} (x_l)^{0.6}$$

Profit Maximization in the Long Run

- In the long run the level of inputs is variable:

$$\max\left(100(x_o)^{0.2}(x_l)^{0.6} - 70x_l - 1x_o\right)$$

- First order conditions:

$$(100)0.6(x_o)^{0.2}(x_l)^{0.6-1} = 70$$

$$(100)0.2(x_o)^{0.2-1}(x_l)^{0.6} = 1$$

Profit Maximization in the Long Run

- First order conditions:

$$(100)0.6(x_o)^{0.2}(x_l)^{0.6-1} = 70$$
$$(100)0.2(x_o)^{0.2-1}(x_l)^{0.6} = 1$$

- Divide first equation by second:

$$3\frac{x_o}{x_l} = 70$$

Profit Maximization in the Long Run

- First order conditions:

$$(100)0.6(x_o)^{0.2}(x_l)^{0.6-1} = 70$$

$$x_o = \frac{70}{3} x_l$$

- Replace second equation into first:

$$x_l = \frac{70}{3}\left(\frac{6}{7}\right)^5 \approx 11$$

Profit Maximization in the Long Run

- Find both inputs:

$$x_l = \frac{70}{3}\left(\frac{6}{7}\right)^5 \approx 11 \qquad \text{consultants}$$

$$x_o = \frac{70}{3}11 \approx 257 \qquad \text{square feet}$$

Profit Maximization in the Long Run

Find output and profits:

$$y = (257)^{0.2}(11)^{0.6} \approx 13$$

$$\Pi = 100y - (70)11 - 257 = 273K$$

Factor Demand Curves

In the example above, the firm chooses inputs so that:

$$pMP_L(x_o, x_l) = salary$$

$$pMP_O(x_o, x_l) = rent$$

Factor Demand Curves

Solve these equations to get factor demand curves:

$$x_l = D_l(salary)$$

$$x_o = D_o(rent)$$

Inverse Factor Demand Curves

Express what the factor price would have to be for factor demand to be at a certain level:

$$salary = f_l(x_l)$$

$$rent = f_o(x_o)$$

Find Factor Demand Curve in the Example

- From first order conditions:

$$(100)0.6(x_o)^{0.2}(x_l)^{0.6-1} = w_l$$

$$x_o = \frac{w_l}{3}x_l$$

- Solve for x_l:

$$x_l = \frac{60^5}{3}\frac{1}{w_l^{\;4}}$$

Find Inverse Factor Demand Curve in the Example

- Solve for w_l :

$$x_l = \frac{60^5}{3}\frac{1}{w_l{}^4}$$

- Get:

$$w_l \approx \frac{127}{x_l{}^{0.25}}$$

Diagram of Inverse Factor Demand for Consultants