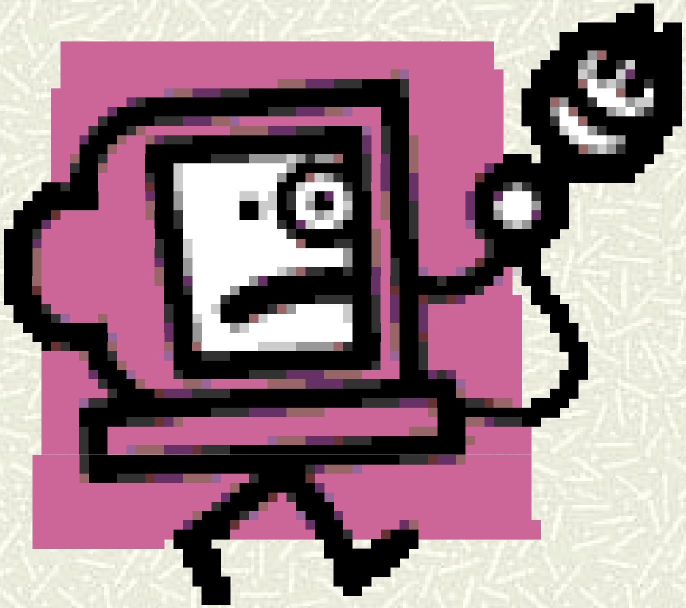
Technology



- Production functions
- Short run and long run
- Examples of technology
- Marginal product
- Technical rate of substitution
- Returns to scale

Analogies with Consumer Theory

Consumers	Firms
Maximize utility	Maximize profits
Constraint: budget line	Constraint: production function

Factors of Production

- Inputs: labor, land, capital, raw materials
- **Physical** capital: inputs that are themselves produced goods, such as tractors, buildings, computers, etc.
- **Financial** capital: money used to start up or run a business (not an input to production)

Production Function

- Function that describes the **maximum** amount of **output** that can be produced for a given level of **inputs**
- Inputs and outputs are measured in **flow units**.
- E.g., with labor, L, and capital, K, as inputs:

$$Y = F(K, L)$$

Technology, Flexibility, and Returns to Scale

Input Flexibility

- How flexible is a firm's technology?
- To obtain a particular output is it possible to substitute one input for another?
- At what rate?

Changing Scale of Operations

- If a firm doubles all inputs, what happens to output?
- Returns to scale.

Short Run and Long Run

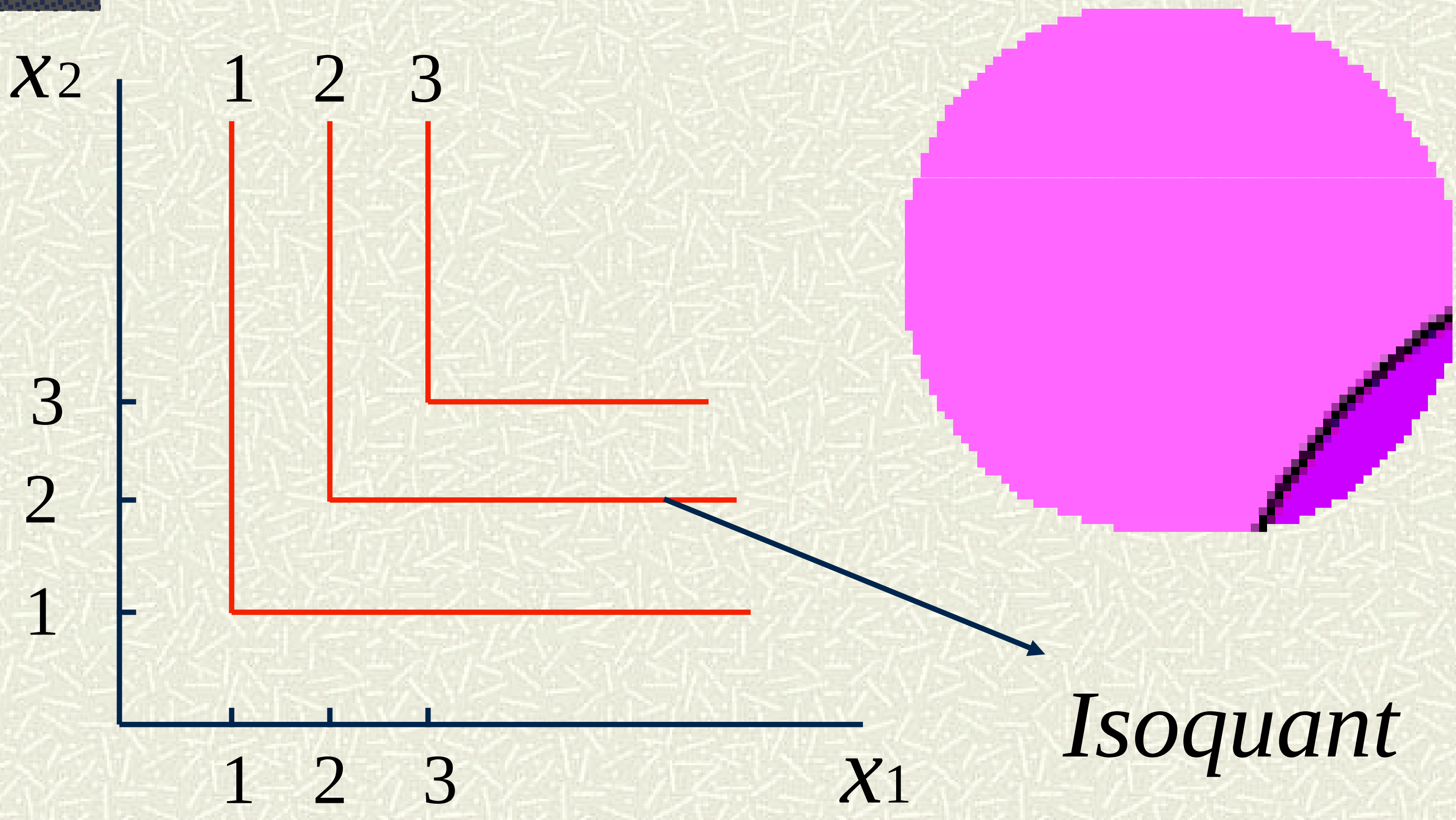
- **Short run**: some factors of production are fixed at predetermined levels.
- Example: in short run, a firm cannot easily vary the size of a plant. It can use machines more intensively.
- E.g.:

$$Y = F\left(\overline{K}, L\right)$$

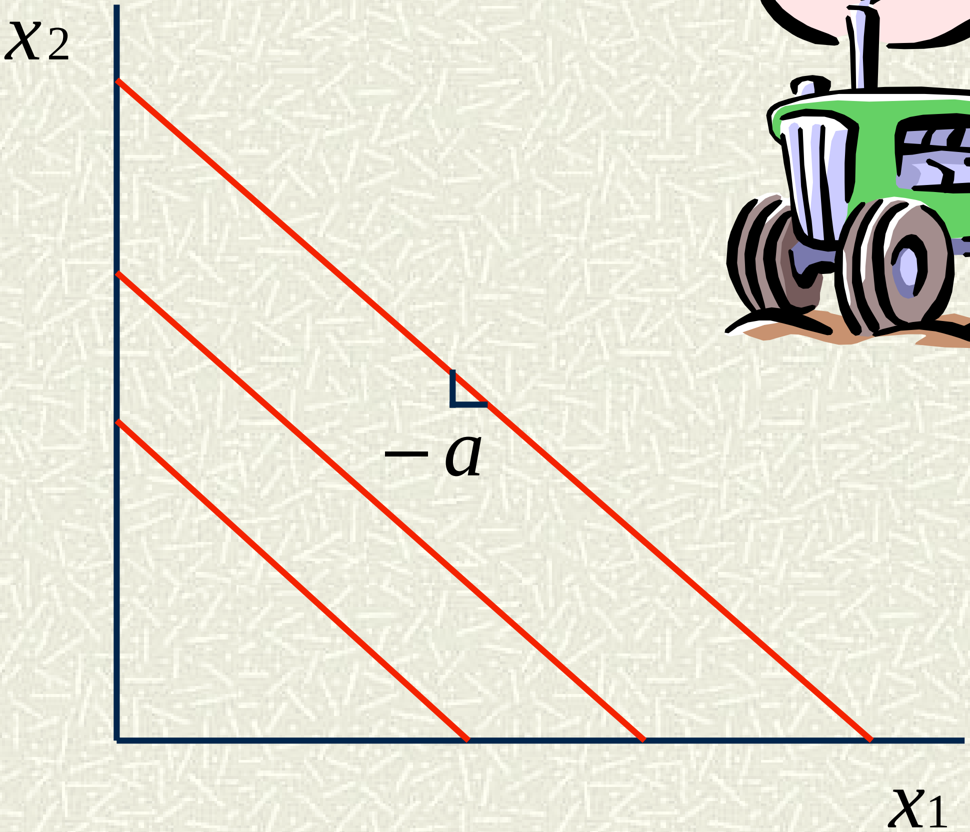
Short Run and Long Run

- **Long run**: all factors of production can be varied. No fixed factors.
- How long is the long run? It depends on the specific type of production.
- For automobile manufacturer it can take years to change size of plant. For travel agency months.

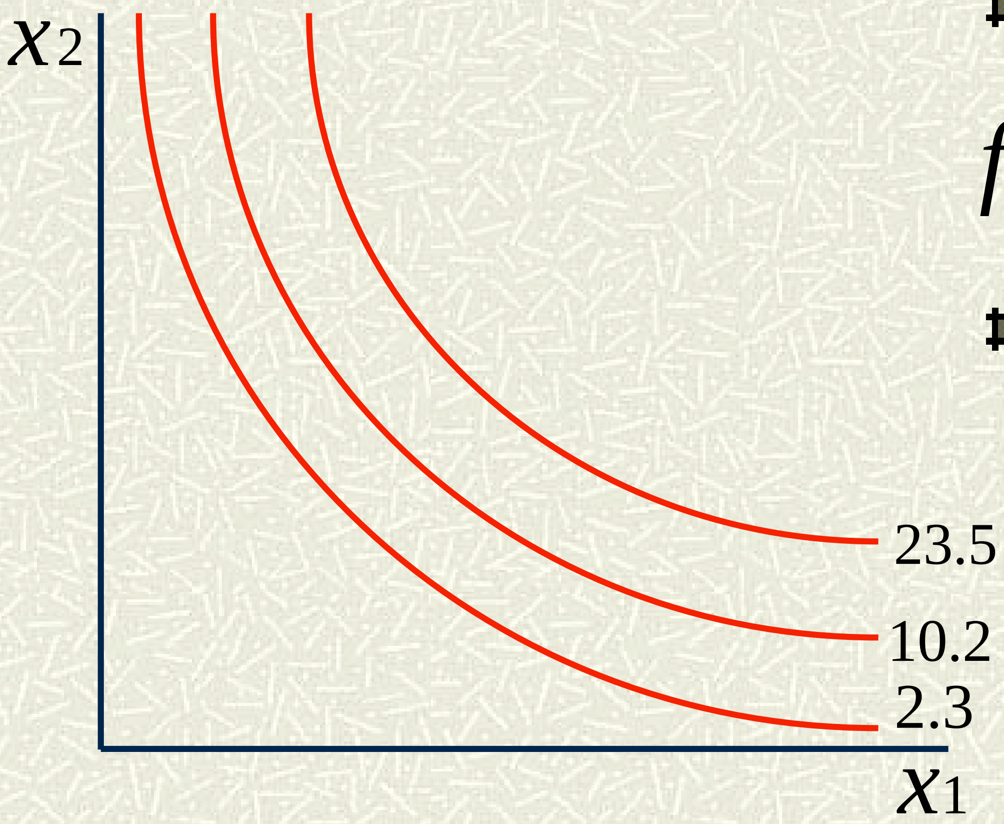
Fixed Proportions: $y = \min(x_1, x_2)$



Perfect Substitutes: $y = ax_1 + x_2$



Cobb-Douglas



- Production function:

$$f(x_1, x_2) = A x_1^{a} x_2^{b}$$

- Isoquants:

$$x_2 = \frac{y^{\frac{1}{b}}}{\left(A x_1^{a}\right)^{\frac{1}{b}}}$$

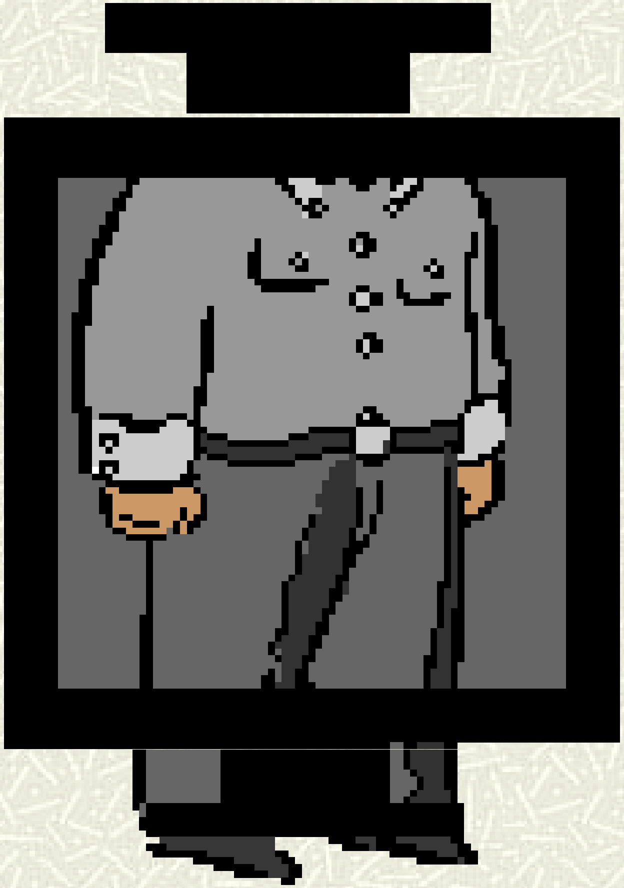
The Marginal Product

- Consider a firm using inputs (x_1, x_2)
- **Q**: by how much is output going to increase if the firm uses "a bit" more of input 1, while keeping input 2 fixed?
- **A**: Marginal product of factor 1:

$$\frac{\partial f(x_1, x_2)}{\partial x_1}$$

The Marginal Product

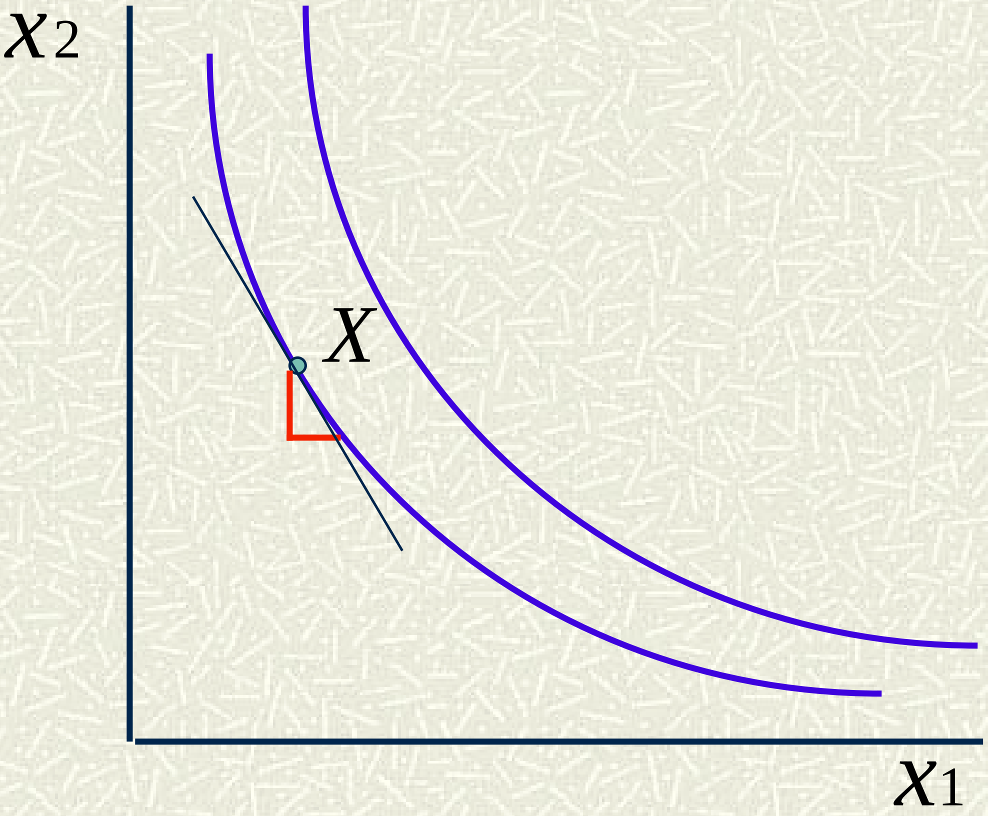
- Typical assumption in economics: the marginal product of a factor **decreases** as we use **more** and **more** of that factor
- E.g.: nurses producing radiographies using given machine



The Marginal Product: Cobb-Douglas

- Consider Cobb-Douglas production function in K and L: $Y = AK^a L^b$
- Marginal product of capital: $\dfrac{\partial Y}{\partial K} = aAK^{a-1}L^b$
- Marginal product of labor: $\dfrac{\partial Y}{\partial L} = bAK^a L^{b-1}$

Technical Rate of Substitution



- TRS is the slope of an isoquant at a given point X
- Measures the rate at which the firm has to substitute one input for another to keep output constant

Computing the TRS

- **Q**: What is the slope of an isoquant (TRS)?
- **A**:

$$TRS(x_1, x_2) = -\frac{\dfrac{\partial f(x_1, x_2)}{\partial x_1}}{\dfrac{\partial f(x_1, x_2)}{\partial x_2}}$$

Computing the TRS: Cobb-Douglas

Cobb-Douglas case:

$$Y = AK^{a}L^{b}$$

$$TRS(K, L) = -\frac{aK^{a-1}L^{b}}{bK^{a}L^{b-1}} = -\frac{a}{b}\frac{L}{K}$$

Returns to Scale

- **Q**: What happens to output when you double the amount of all the inputs?
- **A1**: If output doubles, the technology exhibits **constant returns to scale**.
- E.g.: travel agency that uses office space and travel agents as inputs.

Increasing Returns to Scale

- **A2**: If output more than doubles, the technology exhibits **increasing returns to scale**.
- E.g.: oil pipeline. Double diameter and quadruple cross-section of the pipe.

Increasing Returns to Scale

- If there are increasing returns, it is economically advantageous for a **large** firm to produce, rather than many **small** firms.
- Issue of monopoly.

Returns to Scale

- **A3**: If output less than doubles, the technology exhibits **decreasing returns to scale**.
- Applies to firms with large-scale operations where it is more difficult to coordinate tasks and maintain communication between managers and workers.

Cobb-Douglas and Returns to Scale

- Cobb-Douglas: $f(x_1, x_2) = Ax_1^{a} x_2^{b}$

- Scale inputs by a factor $t > 1$:

$$f(tx_1, tx_2) = A(tx_1)^{a}(tx_2)^{b} = t^{a+b} A x_1^{a} x_2^{b}$$

Cobb-Douglas and Returns to Scale

- Scale inputs by a factor $t > 1$:

$$f(tx_1, tx_2) = A(tx_1)^a (tx_2)^b = t^{a+b} A x_1^{\,a} x_2^{\,b}$$

- If $a + b > 1$, then $t^{a+b} > t$:

$$f(tx_1, tx_2) > tA x_1^{\,a} x_2^{\,b}$$

Cobb-Douglas and Returns to Scale

- Scale inputs by a factor $t > 1$:

$$f(tx_1, tx_2) = A(tx_1)^a (tx_2)^b = t^{a+b} A x_1^{\ a} x_2^{\ b}$$

- If $a + b = 1$, then $t^{a+b} = t$:

$$f(tx_1, tx_2) = tAx_1^{\ a} x_2^{\ b}$$

Cobb-Douglas and Returns to Scale

- Scale inputs by a factor $t > 1$:

$$f(tx_1, tx_2) = A(tx_1)^a (tx_2)^b = t^{a+b} A x_1^{\,a} x_2^{\,b}$$

- If $a + b < 1$, then $t^{a+b} < t$:

$$f(tx_1, tx_2) < tAx_1^{\,a} x_2^{\,b}$$