Lecture 2

Finance Project

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Summary of Mortgages

- MB_t: (mortgage balance remaining at time t)
  MP: (mortgage payment)
  c: (simple monthly interest rate (annual interest rate/12))
  n: (number of months)
- *Mortgage Payments*

$$MP = MB_0 \frac{c(1+c)^n}{(1+c)^n - 1}$$

- *Mortgage balance*

$$MB_t = MB_0 \frac{(1+c)^n - (1+c)^t}{(1+c)^n - 1}$$

Summary of Mortgages (Contd)

- *Scheduled Principal Repayments*

$$P_t = MB_0 \frac{c(1+c)^{t-1}}{(1+c)^n - 1}$$

- *Interest for month t*

$$I_t = cM_{t-1}$$

$$MB_0 \frac{c\left[(1+c)^n - (1+c)^{t-1}\right]}{(1+c)^n - 1}$$

- Check that $MP = I_t + P_t$.

Adjustable Rate Mortgages (ARMs)

- **Index**
  Index at time time t denoted by $x(t)$.
  The interest rate of a mortgage varies according to this contractually specified index. The two most widely used indices are cost of funds index (COFI) and a constant maturity (one year or five year) Treasury index.

ARMs (Contd)

- **Margin**
  Denote by m
  This is the amount the interest rate will be changed by. Margin is also specified in the contract. The size of the margin reflects the value of the various options embedded in the ARM, including the option to prepay, as well as the costs of servicing a loan.
- **Adjustment Period**
  This is the stipulated frequency at which the interest rate will be adjusted. Typical adjustment periods are six months or one year.

ARMs (Contd)

- **Lifetime cap**
  Denoted by c_L.
  This is a contractually specified upper bound on the interest rate.

- **Lifetime floor**
  Denoted by c_F
  This is a contractually specified lower bound on the interest rate.

- **Periodic cap**
  Denoted by c_P
  In a single adjustment period cannot adjust the interest rate by this cap.

Formula for adjusting

- Assume that time is t and we are going to adjust the mortgage interest rate (or coupon rate), which is $c(t-1)$.

- If $x(t) + m > c(t-1)$, then the formula is:

$$\min\left[x(t) + m, c_L, c(t-1) + c_P\right]$$

- If $x(t) \leq c(t-1)$, then the formula is:

$$\max\left[x(t) + m, c_F, c(t-1) - c_P\right]$$

Structure of the document

- Describe the cash-flow of the financial instrument (use equations along with explanations).

- Provide scenarios, or examples of cash-flows.

- Provide a summary of all the formulas. In a real system there will be a formal document describing all the user requirements.

Structure of the document (Contd)

- Also pay attention to extraneous factors (not directed related to the cash-flow).

- We will see examples of this later in the context of MBSs.

Who should be able to understand it

- A user with basic finance knowledge (not familiar with that domain) should be able to understand it.

- Minimize ambiguities.

- The summary is meant for developers and designers.

Mortgage Backed Securities (MBSs)

- The underlying asset for MBSs is a pool of mortgages.

- The cash-flow for MBSs consist of three components
  - Interest.
  - Scheduled Principal Repayment.
  - Payments in excess of the regularly scheduled principal repayment.
- Third component is what makes the cash-flows of MBSs tricky.
- What happens without the third component?

Mortgage Pass-Through Securities

- These are the simplest of MBSs. We will call them *pass-throughs* from here on.

- We will illustrate the cash-flow of a pass-through security through an example and later provide the formulas.

Pass-Throughs (Contd)

- Suppose the pool of mortgages underlying the pass-through has 10 mortgages.

- Each mortgage is worth 100,000 dollars. Total value is 1 million dollars.
- Assume there are 40 pass-throughs issued along with this pool of mortgage.
- Each pass-through is worth 25,000 or 2.5% of the entire amount.

Cash flow for each pass-through

- Each pass-through security gets 2.5% of the cash-flow each month.

- Basically an owner of the pass-through is a *virtual loaning agency* for the pool of mortgages.

- Pass-throughs basically provide liquidity to the mortgage market.

Digression on pass-throughs

- There is always the risk that there might be defaults in the pool of mortgages underlying the pass-through.

- Fortunately majority of pass-throughs are guaranteed by government-related entities:
  - Government National Mortgage Association (Ginnie Mae).
  - Federal National Mortgage Association (Fannie Mae).
  - Federal Home Loan Mortgage Corportaion(Freddie Mac).

Digression on pass-throughs (Contd)

- *Outcome:* Don't have to consider risk of default.

- Notice how extraneous factors are driving our assumptions.

- For pass-throughs backed by private agencies this assumption is lest justified.

Collateralized Mortgage Obligations (CMOs)

- Each pass-through security is exposed to the risk of prepayment uniformly.

- We might want differing risk-levels.

- Again take the same pool of mortgages as before.

CMOs (contd)

- Suppose we have three classes A, B, and C.
- Class A has par value 400,000.
- Class B has par value 300,000.
- Class C has par value 300,000.

CMOs (Distribution of cash-flows)

- Remember the cash flow of the pool of mortgages has two components: *interest* and *principal*.
- Interest is distributed among the three classes proportional to the par value, e.g, class A receives 40% of the interest payments.
- All principal payments go to class A until the mortgage balance reaches 600,000.
- While the mortgage balance is between 600,000 and 300,000, the principal payments go to class B.
- The rest of the time the principal payments go to class C.

CMOs (Contd)

- CMOs described earlier have a very simple rule for prioritizing the distribution principal.

- There are much more complicated schemes such as:
  - Planned amortization class (PAC) bonds.
  - Targeted amortization class (TAC) bonds.
  - Very accurately determined maturity (VADM) bonds.
- These different kind of instruments provide different kind of prepayment risks.

Stripped MBS

- CMOs specify a set of rules for prioritizing the distribution of the principal payments among the various classes (also called *tranches*).

- A stripped MBS has only two classes: *principal-only* or *PO* class, and *interest-only* or *IO* class.

- PO class only receives the *principal*.

- IO class only receives the *interest*.

Cash flow characteristics of CMOs

- MP_t mortgage payment for month t.

- MB_{t-1} mortgage balance at the start of month t.

- I_t (monthly interest form month t).
- NI_t (net interest for month t).
- S_t (servicing fee for month t).
- s (rate of servicing fee).
- P_t (principal payment form month t).
- PP_t (prepayment for month t).

Cash flow (contd)

We have following relationships between the quantities:

$$
\begin{aligned}
MP_t &= MB_{t-1}\frac{c(1+c)^{n-t+1}}{(1+c)^{n-t+1}-1} \\
I_t &= cMB_{t-1} \\
NI_t &= (c-s)MB_{t-1} \\
S_t &= sMB_{t-1} \\
MB_t &= M_{t-1} - P_t - PP_t
\end{aligned}
$$

Pass-Throughs

- Assume that we have n pass-throughs and the underlying mortgage pool has the cash-flow at time t of

$$CF_t = NI_t + P_t + PP_t$$

- Cash flow for each pass-through is simply $\frac{C_t}{n}$.

- Cash-flow divided evenly between each pass-through security.

CMOs

- The interest component of the cash-flow is NI_t.

- Assume we have k classes or tranches $T_1, \cdots, T_k$ and w_i is the weight of tranch T_i.

- Weight w_i is the *par value* of that tranch divided by the total value of the mortgage pool.

- Tranch T_i receives $w_i NI_t$ from the interest rate component.

CMOs

- Once a tranch receives principal payments equal to its par value it is said to *retire*.

- The principal component of the mortgage payment is $PP_t + P_t$.

- Let T_i be the tranch with the highest priority that has not retired.

- Pay the principal payment to that tranch.

Modeling Prepayment

- Prepayment is affected by several factors. We will list some of them here.
- *Refinancing opportunities*
  Prevailing refinancing rates. Lower refinancing rates relative to the mortgage coupon/interest rate leads to higher likelihood of prepayment.
- *Age or seasoning*
  The age of a mortgage affects prepayment decisions. Newly originated mortgages are unlikely to be prepaid, while aged mortgages, for example, held by retirees, are more likely to be prepaid.

Modeling Prepayment (Contd)

- *Seasonaility*
  The time of year affects prepayments. Spring and summer months tend to experience greater prepayment activity.
- *Burnout*
  Borrowers in a pool are heterogenous in their response to refinancing opportunities. Once the eager borrowers have refinanced the rest of the borrowers (the laggards) will not prepay for a while. This phenomenon is called *burnout*.

Simple Model of prepayment

- Forecasting prepayments is crucial to computing the cash-flow of MBSs.

- Current practice is to use Public Securities Association (PSA) prepayment benchmark.

- In this simple model a fraction of the principal is assumed to be prepayed each month.

SMM and CPR

- *Conditional Prepayment Rate* (CPR) is the annual prepayment rate.
- CPR is based on the characteristics of the pool and the current expected economic environment.
- *Single Monthly Mortality rate* (SMM) is the fraction of the principal payed off each month, and is related to CPR in the following way:

$$SMM = 1 - (1 - CPR)^{\frac{1}{12}}$$

- Check that the following formula is true:

$$P(1 + SMM)^{12} = P(1 - CPR)$$

Prepayment and SMM

- Let MB_{t-1} be the mortgage balance at time $t - 1$ and SP_t the servicing fee for month t.

- The prepayment for month t is given by the following formula:

$$PP_t = SMM_t(MB_{t-1} - S_t)$$

- Recall that S_t is the servicing fee for that month.

PSA benchmark

- PSA standard benchmark assumes the following prepayment rates for 30-year mortgages:
  1. A CPR of 0.2% for the first month, increased by 0.2% per month for the next 30 months, when it reaches 6% year.
  2. A 6% CPR for the remaining years.
- The standard PSA benchmark (written as 100% PSA) can be expressed using the following formula:

$$CPR = \begin{array}{ll} \frac{6\%t}{30} & \text{if } t \leq 30 \\ 6\% & \text{if } t > 30 \end{array}$$

PSA (contd)

- A x% PSA varies the speed of the prepayment by $\frac{x}{100}$.
- A 50% will assume one-half the CPR of the standard PSA.
- A 150% will assume 1.5 times the CPR of the standard PSA.

References

- F.J. Fabozzi, Fixed Income Mathematics (Analytical and Statistical Techniques), Third Edition, *Irwin Professional Publishing*, 1997.
- W.N. Torus, Mortgage Backed Securities, *Handbooks in Operations Research and Management Science (Volume 9), Editors: R.A. Jarrow, V. Maksimovic, W.T. Ziemba*, Informs, North-Holland, 1995.

Scenarios

- Pass-through security with original mortgage balance of 100 million dollars.
- Mortgage rate is 9.5%.
- Servicing fee is 0.5%.
- 360 months to maturity.
- Prepayment according to 100% PSA.

Parameters

- $MB_0 = 100,000,000.$
- $n = 360.$
- $c = .0079167.$
- $s = .0004167.$
- CPR_t is given by the following formula:

$$\begin{array}{ll} \frac{6\%t}{30} & \text{if } t \leq 30 \\ 6\% & \text{if } t > 30 \end{array}$$

Example cash-flows (Check)

- $t = 1$
  $MP_t = 841$, $SP_t = 49$, $I_t = 792$, $PP_t = 17$, $S_t = 42$, $CF_t = 816$

- $t = 100$
  $MP_t = 544$, $SP_t = 25$, $I_t = 475$, $PP_t = 308$, $S_t = 25$, $CF_t = 827$

- $t = 358$
  $MP_t = 144$, $SP_t = 141$, $I_t = 2$, $PP_t = 1$, $S_t = 0$, $CF_t = 142$

150% PSA

- CPR_t given by the following formula:

$$\begin{array}{ll} \frac{9\%t}{30} & \text{if } t \leq 30 \\ 9\% & \text{if } t > 30 \end{array}$$

50% PSA

- CPR_t given by the following formula:

$$\begin{array}{ll} \frac{3\%t}{30} & \text{if } t \leq 30 \\ 3\% & \text{if } t > 30 \end{array}$$

Drawbacks of PSA

- The PSA model is very simple. It is a benchmark and is simply a market convention.

- Does not model *seasonal* or *burnout* effects.

Vector of PSAs

- One could *vary the speed* of the prepayment for different time.
- For example,
  - Months 1-43 (100% PSA)
  - Months 44-204 (150% PSA)
  - Months 205-360 (50% PSA)
- Speed of prepayment could also depend on mortgage balance.
- Probably can handle seasonal effects in this manner.

More complicated prepayment models

- Assume that the number of prepayments in month t (denoted by pn_t) follows a Poisson distribution with the parameter $\lambda(X_t, \beta)$.

- Let N_t be the number of mortgages in the pool.

- In this case expected number of prepayments is given by:

$$N_t \lambda(X_t, \beta)$$

- X_t set of variables to be defined.

- β parameters to be defined.

Form of λ

- *Refinancing opportunities*
  $rf7$
  The ratio of 7 year Treasury bond to the mortgage pool's weighted-average-coupon (WAC).
- *Age or Seasoning*
  $\pi_0(t)$
  The ratio of *old* mortgages to the *new* mortgages at time t.

Form of λ (Contd)

- *Seasonal affects*
  $season$
  Discrete variable indicating which season we are in. For example, $season = 1$ for months of April through September and zero every where else.
- *Burnout*
  $burnout$
  Ratio of the pools' current principal and the principal if the mortgage pool had been payed off according to a standardized benchmark (say 100% PSA).

Form of λ

- $\lambda(X_t, \beta)$ is given by the following equation:

$$\pi_0(t) \exp(\beta_1 \cdot rf7 + \beta_2 \ln(burnout) + \beta_3 \cdot season)$$

- How does one estimate the parameters $\beta_1, \beta_2, \beta_3$.

- Use Maximum-Likelihood-Estimation. Will be covered in a statistics course.

Requirements Document

- Due by: Wednesday, Jan 27, 1999.
- Describe the financial instruments and the underlying assets in great detail. Equations for cash-flows.
- Example of cash-flows (*scenarios*).
- Summary of formulas.

Forming sub-teams

- Divide your team into the following sub-teams. Everyone should be involved in some role.
- *Researchers*
  Students in this sub-team will actually do the research and write down the equations describing the cash-flows of the instruments and the assets. Mention your resources very clearly.

- *Scenario builders*
  These students will look at the document provided the researchers and provide examples of cash-flows.

Forming sub-teams (Contd)

- *Reviewers*
  The document produced by the researchers and the scenario-builders will be reviewed and a summary will be written by this sub-team. This sub-team will also make sure that everything fits together.

Document

- Mention the composition of the sub-teams on the top page.

- Use a standard format to compose the document.

- Researchers should mention their sources very clearly.

Penalty

- If you are late by more than 2 days, 10% deducted for every 2 days.
- Be on time. We will provide feedback on your document in a week or so.
- Mention all the background for the assets and the financial instruments.