Lecture 1
Finance Project
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Goals of the course

- Teach the students to systematically design algorithms and systems for problems in the finance domain.

- Taking finance ideas from the academic domain and making them real.

- **Maxim:** Learn by doing.
Nature of the course

• Largely independent in nature.

• Jeff and I are here to guide you, but you will work as independent teams.

• Think of the course as a structured independent study course.
What we will do in class

• Jeff and I will go through a systematic design of a system for pricing mortgage backed securities or MBSs.

• Use the lectures as a guide for your project.

• There will be five phases to the project. These phases will be defined later.
Logistics

- Students will form a team of 3-4.

- Each team will select a paper from a set provided in class.

- Each team will design a system based on the paper they select.

- Each team will go through the five phases (to be described later).
Some useful tips

• Pick a balanced team.

• Stick to the schedule for each phase.
Grading

• Grading will be done depending on the outcome of each phase.

• No tests and homeworks.
Five phases

Description of these phases will be provided later, but here they are.

- Requirements phase (*Phase 1*).

- High-level design phase (*Phase 2*).

- Low-level design phase (*Phase 3*).
Phases (Contd)

- In class presentation (Phase 4).

- Prototype (Phase 5)
Prototype

• This will be a *scaled back* version of the design.

• Make as many limiting assumptions as possible, but state them carefully.

• You can use **C, C++, JAVA** to implement the prototype. Make sure you tell Jeff the environment you are using.
Presentation

• There will be one presentation per team.

• The presentation will be a synopsis of phase 1, 2, and 3.

• Each presentation will be 20 minutes long.

• Last three lectures will be all presentations.
Paper 1

Paper 2

Paper 3

Paper 4

Paper 5

Paper 6

Paper selection

• Don’t choose papers 5 or 6 unless you are comfortable with stochastic calculus.

• Focus on the techniques and algorithms in the paper. It is OK if you don’t understand all the mathematical derivations.
Goals of reading the paper

• Decide what financial instruments you want to price after reading the paper. Pick 2-3 instruments. You will be required to understand these instruments completely.

• You should have a clear idea about the algorithm proposed in the paper.

• Make a note of advantages/disadvantages of this technique/algorithm.
Requirements document

- Describe the financial instrument in great detail.

- Describe the assets the instruments depend upon. State the assumptions on the prices of these assets. Describe the cash-flow characteristics.

- Describe the financial instruments and there cash-flow characteristics.
Requirements document

- In an abstract sense we are describing what is the semantics of each operation that the user can do.

- In this very specific example this amounts to defining the precise semantics of mortgage backed securities (MBSs).
Describing Mortgages

• *Fixed Rate*: The annual interest rate of the mortgage stays fixed throughout the life of the mortgage.

• *Adjustable Rate Mortgages (ARMS)*: The annual interest rate can be adjusted by the loaning agency.
Fixed Rate Mortgages

- Let $MB_0$ be the original mortgage balance.

- Let $c$ be the simple monthly interest rate.

- Let $MP$ be the monthly mortgage payment.

- Let $n$ be the number of months.
Relationship between $MP$ and $MB_0$

- The following equation should hold between $MP$ and $MB_0$:

$$MB_0 = MP \sum_{i=1}^{n} (1 + c)^{-i}$$

$$= MP \frac{1 - (1 + c)^{-n}}{c}$$

- Hence the monthly mortgage payment $MP$ is given in terms of the mortgage amount $MB_0$ using the following formula:

$$MP = MB_0 \frac{c(1+c)^n}{(1+c)^n - 1}$$
Principal at time $t$

• Let $MB_t$ the remaining mortgage balance at time $t$.

• We have the following relationship between $MB_t$ and $MP$.

\[ MB_t = MP \frac{1 - (1 + c)^{-(n-t)}}{c} \]

• So we have the following equation between $MB_0$ and $MB_t$:

\[ MB_t = MB_0 \frac{(1 + c)^n - (1 + c)^t}{(1 + c)^n - 1} \]
Breaking the mortgage payments

- At time $t$ the mortgage balance is $MB_{t-1}$ ($t \geq 1$).

- The interest $I_t$ on this is mortgage balance is:

  $$cMB_{t-1}$$

- The mortgage payment $MP$ at time $t$ is broken into two parts: interest payment $I_t$ and payment applied towards principal $P_t$. We have the following equation:

  $$MP = I_t + P_t$$
Scenarios

• A requirements document for a large software system has a huge number of scenarios.

• Basically, *scenarios* describe what should happen in specific cases.

• For example, in the requirements document for an online brokerage system a scenario might describe what should happen when a user logs on and buys a stock.
Examples

- In this case, scenarios are simply examples of cash flows.

- Consider a mortgage of 100,000, annual mortgage rate (12c) of 9.5% and time period of 30 years (360 months).

- Check that the monthly mortgage rate $MP$ is 840.85.
Example continued

• Check that $I_0 = 791.67$ and $P_0 = 49.19$.

• Check that $I_{215} = 574.95$ and $P_{215} = 265.90$.

• $I_t$ is a decreasing function of $t$ and $P_t$ is an increasing function of $t$. (Why?).
ARMs

- ARM interest rate can be adjusted by a margin $m$ at a frequency specified in the contract.

- *Lifetime cap* $c_L$: This is an upper bound that the interest rate cannot exceed.

- *Lifetime floor* $c_F$: This is a lower bound on the interest rate.
ARMs

Let us the interest rate is $c(t - 1)$ at time $t - 1$ and we are adjusting at time $t$,

The new interest rate $c(t)$ is given by the following cases:

- if $x(t) + m > c(t - 1)$
  \[
  \min [x(t) + m, c_L, c(t - 1) + c_P]
  \]
- if $x(t) + m \leq c(t - 1)$
  \[
  \min [x(t) + m, c_F, c(t - 1) - c_P]
  \]
Explanation of terms

- $x(t)$: Underlying index specified in the contract. Two widely used indices are cost of funds index (COFI) and a constant maturity (one year or five year) Treasury index.

- $c_L$ and $c_F$ are the lifetime cap and floor respectively.

- $c_P$ denotes the ARMs periodic cap, i.e., cannot adjust by more than this amount.
New Mortgage Payment

• Assume that current time is $t$, the interest rate, mortgage balance at time $t - 1$ are $c(t - 1)$ and $MB_{t-1}$.

• The new adjusted interest rate is $c(t)$.

• The new mortgage payment $MP(t)$ is given by the following expression:

$$MP(t) = MB_{t-1} \frac{1 - (1 + c(t))^{-(n-t)}}{c(t)}$$

• Everything else stays the same.
Action Items

- Pick your teams (3-4 students) and send e-mail to Jeff or me.

- Pick a paper (Norene Mears has a copy of each paper) and copy it. *Return the master copy.*

- Papers should be in London and New York in 2-3 days.

- Read the paper and decide on the financial
instruments you are going to price.

• Start reading about the financial instruments you are going to price.