Solution to the Chooser Question

The payoff of the chooser option at time 1 is given by $\max[C(S_1,X,1),P(S_1,X,1)]$, where $C(S_1,X,1)$ is the value of a call option that matures at time 2, with exercise price X and stock price S_1 . Put call parity implies that at time 1, $C(S_1,X,1)$ - $P(S_1,X,1)$ = S_1 -X/R. Substituting in for the value of the put, the chooser payoff at time 1 is given by $\max[C(S_1,X,1),C(S_1,X,1)-S_1+X/R] = C(S_1,X,1)+\max[0,X/R-S_1]$. But, $\max[0,X/R-S_1]$ is the payoff of a put option that expires at time 1, with exercise price X/R. So, at time 0, the value of the chooser must be equal to $C(S_0,X,2)$ + $P(S_0,X/R,1)$. We can then use put call parity again to substitute out the value of the one period put in terms of a one period call, giving us the final result that the chooser payoff equals $C(S_0,X,2)$ + $C(S_0,X/R,1)$ - S_0 + X/R^2 .

You would likely purchase the chooser option of you wanted a positive payoff in the tails of the distribution of the underlying return in the future.

To solve for the value of the chooser, we work recursively through the tree. The call option payoffs are given by

$$C_{uu} = Max(15.625-10,0) = 5.625$$

$$C_{u}$$

$$C_{ud} = Max(10-10,0) = 0$$

$$C_{d}$$

$$C_{dd} = Max(6.4-10,0) = 0$$

Clearly, after the first down move, the call is worthless. To value it after the up move, we use the formula from the solution to the third homework. The value of the call will be $C_u=(1/R)[((R-d)/(u-d))C_{uu}+((u-R)/(u-d))C_{ud}]=(1/1.1)[((1.1-0.8)/(1.25-0.8)) 5.625+((1.25-1.1)/(1.25-0.8))0]=3.41.$ So, $C_u=3.41$, $C_d=0$

The put payoffs are given by

$$P_{uu} = \max(10-15.625,0) = 0$$

$$P_{u}$$

$$P_{ud} = \max(10-10,0) = 0$$

$$P_{d}$$

$$P_{dd} = \max(6.4-10,0) = 3.6$$

If the first move is an up, then the put is worthless. To value the call option after the down move, we use the formula given in the solution to the third homework: $P_d=(1/R)[((R-d)/(u-d))P_{du}+((u-R)/(u-d))C_{dd}]=(1/1.1)[2/3(0)+1/3(3.6)]=1.09.$ So, $P_u=0,P_d=1.09.$

The chooser payoff at time 1 is given by 3.41 after an up move and 1.09 after a down move. So, the initial value of the chooser is (1/1.1)((2/3)(3.41)+(1/3)(1.09))=2.40