

## Objectives

- Finish fixed-income
- Options
  - what
  - simple strategies
  - introduction to pricing

## Including Time Value or Putting it all together

- Up to now, ignored time value
- Change in position over small unit of time

$$\begin{aligned}\frac{\Delta P}{P} &\approx y\Delta t - D_p\beta_p\Delta F + 0.5C_p\beta_p^2(\Delta F)^2 \\ &= A_p\Delta t - D_p\beta_p\Delta F + 0.5C_p\beta_p^2(\Delta F)^2\end{aligned}$$

## Portfolio Movements

- Invest  $x$  in 1 year PDB and  $(1-x)$  in 2 year PDB
- 50% in each of the bonds
  - with sensitivities 1.1, and 1.2
- intercepts of 0.001 and 0.002
- set  $\Delta t=1$

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## Computations

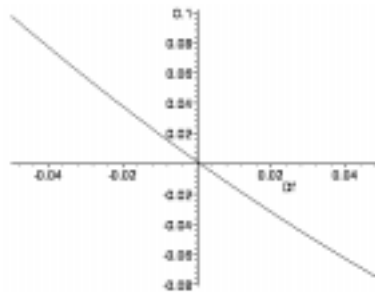
$$\begin{aligned}
 \frac{\Delta P}{P} &\approx (xA_1 + (1-x)A_2) - (xD_1\beta_1 + (1-x)D_2\beta_2)\Delta F \\
 &\quad + 0.5(xC_1\beta_1^2 + (1-x)C_2\beta_2^2)(\Delta F)^2 \\
 &\approx (0.5(0.001) + 0.5(0.002)) - (0.5(1.1) + 0.5(2)(1.2))\Delta F \\
 &\quad + 0.5(0.5(1.1)^2 + 0.5(2)^2(1.2)^2)(\Delta F)^2
 \end{aligned}$$

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## Portfolio Returns



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## Adjusted Convexity vs. Yield

- spread position: short bullet and long barbell
- Modified duration = 0
- Positive convexity
- Intercept?

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# Fixed Income Summary



- PDB prices
- forward rates and no-arbitrage restrictions
- Risk Management
  - duration and convexity for parallel shifts
    - PVBP, modified duration
    - price of convexity
  - adjusting for non-parallel shifts: factor model

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# Introduction to options

- Options are everywhere
  - traded options: basic and exotic
  - interest rate options
  - many securities have embedded options
    - callable debt, convertible debt, rights issues, warrants, mortgages, etc.
  - Many real investments have option features

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## Terminology

- Call
- Put

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## More terminology

- American style
- European style

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## Terminology

- Exercising:
- Strike (exercise) price:
- expiration date

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## More terminology

- In the money
- At the money
- Out of the money

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## Stock Options

- 3 month cycle
- expire on 3rd Saturday of month, trading stocks Friday afternoon
- protected for stock splits, stock dividends, NOT for regular dividends...

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## Exotics

- Asian options
- lookback
- barrier
- binary

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## Notation

- $C_t$ : price of call,  $P_t$ : put price,  $S_t$ : stock price
- $r$ : risk free rate

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## Sequence of events for American call

- Now: purchase call for  $C_0$
- Exercise time,  $T$ 
  - $S_T > X$ : exercise and pay  $X$  for stock
  - $S_T < X$ : don't exercise

$$C_T = \begin{cases} S_T - X, & \text{if } S_T > X \\ 0, & \text{otherwise} \end{cases}$$
$$= \max(0, S_T - X)$$

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## Puts

- Similar timing
- Buying put not the same as writing call

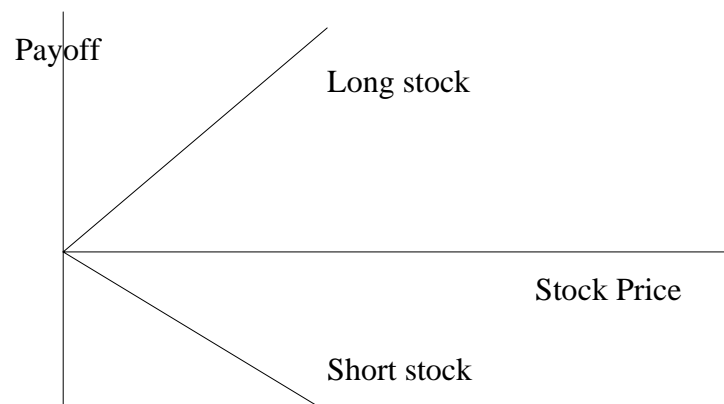
$$P_T = \begin{cases} 0, & \text{if } S_T > X \\ X - S_T & \text{if } S_T \leq X \end{cases}$$
$$= \max(0, X - S_T)$$

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## Payoff Diagrams

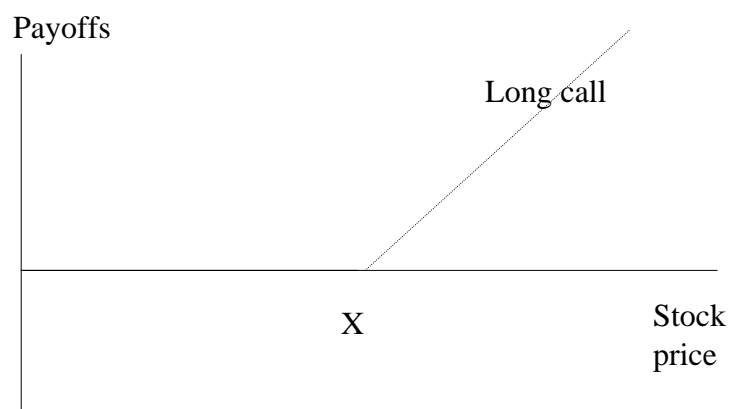


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## European call

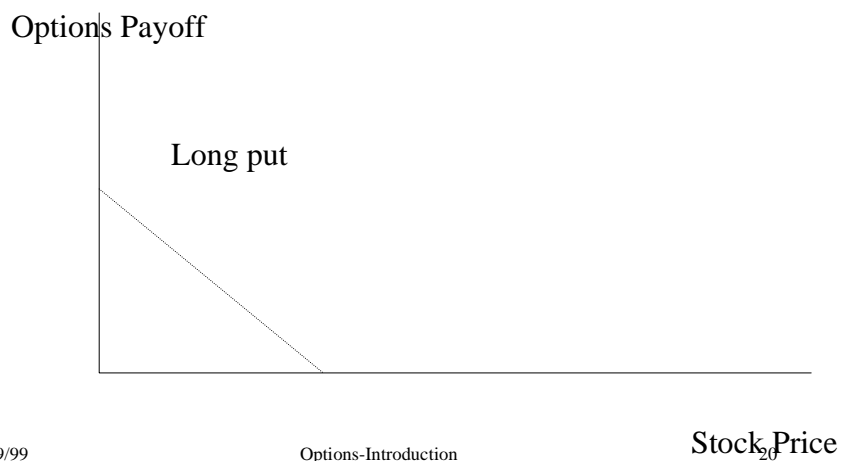


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## European Put



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Stock Price

## Some strategies

- Naked
- Protective put
- Covered call
- Straddle
- Strangle

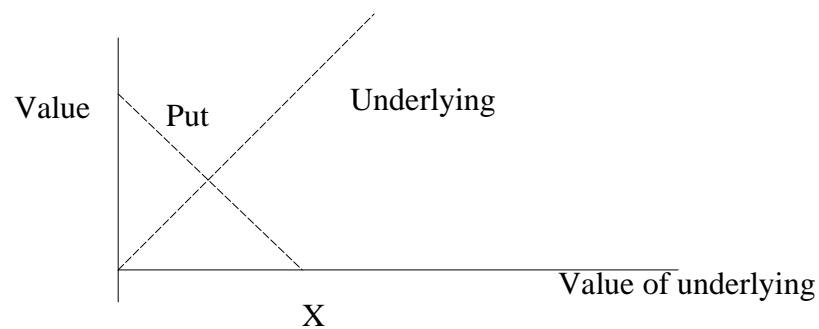
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## Protective put

- Purchase underlying security
- Purchase put option, exercise price  $X$



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## Algebraically

Position	$S_T < X$	$S_T > X$
Underlying	$S_T$	$S_T$
Put	$X - S_T$	$S_T$
Net	$X$	$S_T$

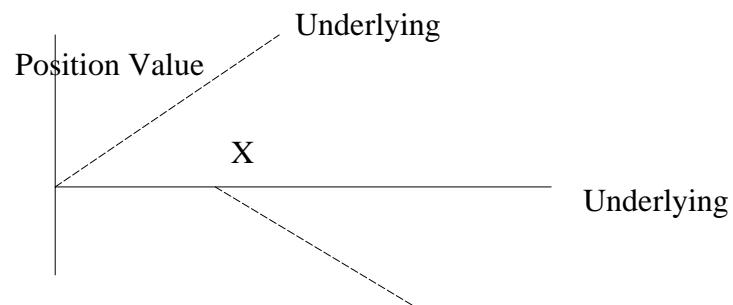
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## Covered call

- Purchase underlying
- Write call option against it



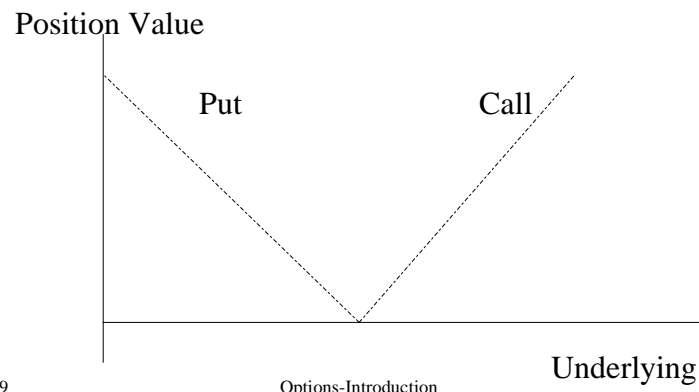
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## Spread

- Buy put and call, both at the same strike



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## Strangle

- Buy call and put, call exercise  $<$  put exercise



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## Spreads

- combination of 2+ calls or puts, same asset with differing exercise prices or times to expiration
  - Vertical or money spread
    - Same maturity and different exercise price
  - Horizontal or time spread
    - different maturities

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## Main Points

- Lots of strategies possible
- Options allow you to customize cash flows across states in the future

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## Put call parity

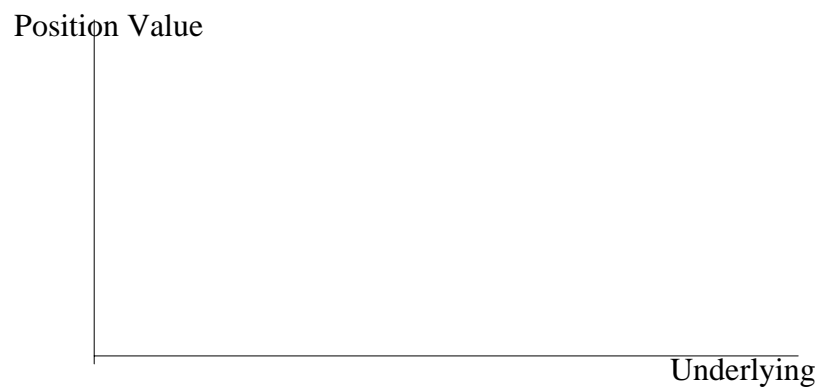
- Relationship between price of European call and put
- Independent of assumptions about randomness in underlying
  - stocks
  - indexes
  - bonds, currencies, etc.

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## Final Payoffs: Long call and short put



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# Arbitrage relationship

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## Summary

- Adding passing of time to bond models
- Options
  - what
  - basic strategies
  - put/call parity

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## Next Time

- More basic properties of option values
- Basic option pricing and hedging model
  - binomial model