

Interest Rate Risk Management

Risk management tools: duration etc.

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Duration

1

Objectives

- Apply duration, PVBP, etc. to trading situations
- Application to Orange County Bankruptcy
- Explain how repo works

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Duration

2

Duration

- % Change in bond price for a % Change in Yield
- Formula:

$$D = -\frac{\partial B}{\partial Y} \times \frac{Y}{B}$$

$$D = \frac{CF_1}{1+Y/B} \times 1 + \frac{CF_2}{(1+Y)^2/B} \times 2 + \dots + \frac{CF_T}{(1+Y)^T/B} \times T$$

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3

Interpretation

- parallel shifts in term structure, return proportional to duration

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4

Example

- Yield = 8%
- 30 year PDB and 1 year PDB
- Durations?

$$b_1 = \frac{1}{1.08} = 0.93$$

$$b_{30} = \frac{1}{1.08^{30}} = 0.0994$$

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5

Yield rise

- New Yield: 1.082
- % Price changes:

$$b_1 = \frac{1}{1.082} = 0.93, \text{ or } 0.185\% \text{ drop}$$

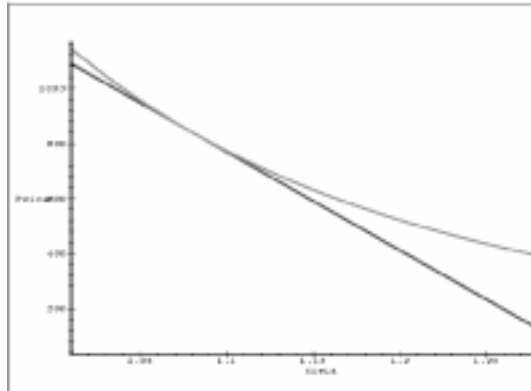
$$b_{30} = \frac{1}{1.082^{30}} = 0.0940, \text{ or } 5.4\% \text{ drop}$$

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6

Bond Price vs. Yield



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7

Example

- Bond with 10% semi-annual, 8 payments left, annualized yield of 9%.
- Aside: in practice semi-annual compounding

$$P = 5 \times \left(\frac{1}{0.045} \right) \times \left(1 - (1.045/2)^{-8} \right) + 100 \times (1.045)^{-8}$$
$$= 103.2979$$

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8

Duration

$$\begin{aligned} D &= -\frac{\partial P}{\partial Y} \times \frac{Y}{P} \\ &= \frac{5/1.045}{103.30} \times 1 + \frac{5/1.045^2}{103.30} \times 2 + \dots + \frac{105/1.045^8}{103.30} \\ &= 6.81 \end{aligned}$$

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9

Bond Returns or So What?

- Suppose yield drops to 8.8%, next day
- New bond price: 103.97, Change: 0.67
- Duration approximation:

$$\begin{aligned} \Delta P &\approx -D \times \frac{\Delta Y}{Y} \times P \\ &= -6.81 \times \frac{0.044 - 0.045}{1.045} \times 103.30 \\ &= 0.67791 \end{aligned}$$

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10

Holding Period Returns

- Buy bond at 9% and sell after yield drop
- Return = $(103.97-103.30)/103.30=0.648\%$

$$\begin{aligned}\frac{\Delta P}{P} &\approx -D \frac{\Delta Y}{Y} \\ &= 6.81 \frac{0.044 - 0.045}{1.0045} \\ &= 0.6526\%\end{aligned}$$

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11

Portfolio duration

- Invest $w_i\%$ of your \$ in bond I
- D_i duration of bond I
- Portfolio duration:
 - $D_p = w_1 D_1 + w_2 D_2 + \dots + w_I D_I$

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12

Example

- Invest half your \$ in a 2 year PDB and the rest in a 30 year PDB
- Portfolio duration?

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Duration

13

Duration Application: Immunitization

- Match duration of liabilities and assets
- Why?

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14

Immunization

- You have a liability of 100 in 2 years
- Duration?
- Suppose you use 1 year PDB and previous coupon bond to form immunized portfolio?
- Current yield is 9%

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15

Strategy

- Invest $x\%$ in one year PDB and $(1-x)\%$ in coupon bond

$$\begin{aligned}4 &= xD_{1year} + (1-x)D_{coupon} \\ &= x2 + (1-x)6.81 \\ \rightarrow x &= \frac{2.81}{4.81} = 0.5842\end{aligned}$$

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16

Position

$$V_{liability} = \frac{100}{1.045^4} = 83.85613$$

$$P_{pdb} = \frac{100}{1.045^2} = 91.573$$

$$\# \text{ PDB} = \frac{0.5842 \times 83.86}{91.573} = 0.535$$

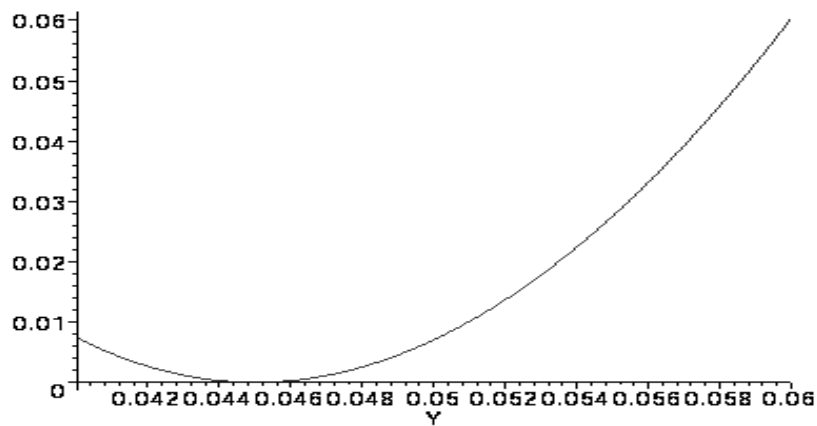
$$\# \text{ Coupon} = \frac{0.4156 \times 83.86}{103.30} = 0.338$$

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17

Portfolio Value vs. Yield



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18

Modified Duration

- % Change of bond price for given yield change
- Units of percentage returns
- same info as duration

$$\begin{aligned}MD &= -\frac{\partial P}{\partial Y} \frac{1}{P} \\ &= D \times \frac{1}{Y} \\ &= 6.81 \times \frac{1}{1.045}\end{aligned}$$

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19

PVBP

- Change in price for change in basis point
- Units: \$ per bond

$$\begin{aligned}PVBP &= -\frac{\partial P}{\partial Y} / 100 \\ &= D \frac{P}{Y} / 100 \\ &= 6.81 \times \frac{103.30}{1.045} / 100\end{aligned}$$

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20

Using PVBP to hedge

- 2 bonds: 2 year and 20 year
- currently long 1 20 year bond, and want to hedge with 2 year

$$n_{20}PVBP_{20} + n_2PVBP_2 = 0$$

$$n_2 = -n_{20} \frac{PVBP_{20}}{PVBP_2}$$

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21

Speculation

- Suppose 20 year yield > 2 year yield
- Expect spread to widen
- What should you do?

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22

Orange Country

- Dec 1994, \$1.6 **billion** loss
- largest municipality to declare bankruptcy
- Treasurer: Bob Citron, \$7.5 investor pool
- Pool: Orange County Investment Pool
- Customers
 - country, city, schools

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Duration

23

What was going on?

- Duration!
- Citron previously made 2% higher returns than State pool
- local schools issued ST notes to invest in pool
- 1994 election, opponent pointed out problems

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24

What happened

- 1994 Fed increased rates
 - big paper losses in pool
 - bankruptcy declared
 - losses realized

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Duration

25

Citron's strategy

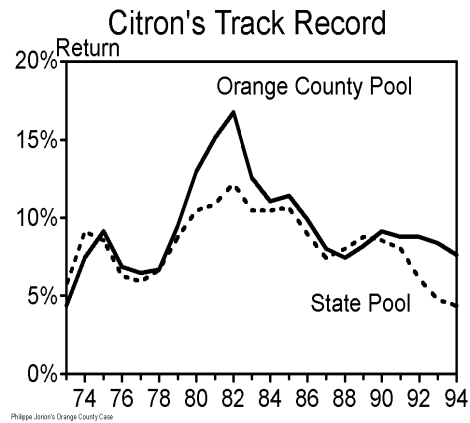
- Upward sloping TS
- Dec 1993
 - short term yields < 3%
 - 5 year yields roughly 5.2%
- after liquidation, 2.5% interest rate drop
- borrowed short and invested long
- levered too!

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26

Returns

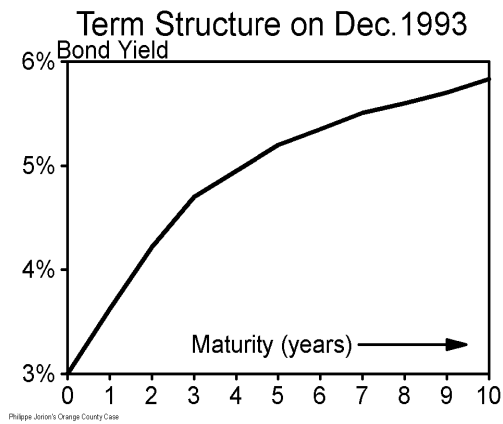


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Duration

27

Before Bankruptcy



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Duration

28

How to increase duration

- Increase duration in 2 ways:
 - invest in longer bonds
 - leverage

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29

Example

- \$100 5 year note
- use as collateral for cash in reverse repo
- use cash to buy new note,
- continue...
- If duration of notes is 4 and you do this 3 times?

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Duration

30

Reverse Repo Transaction

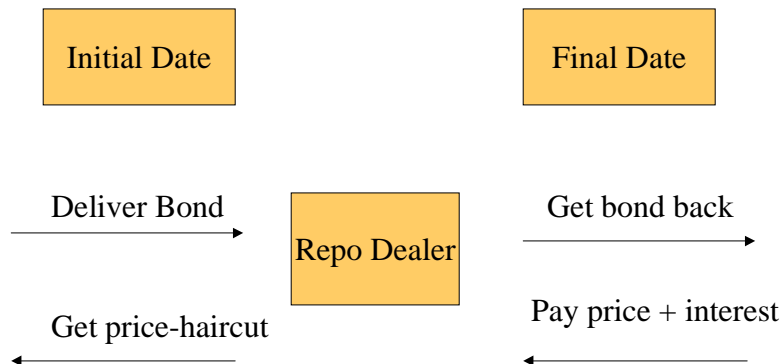
- Short term borrowing or lending using treasuries as collateral
- very common
- give your bonds to dealer for \$ today
- commit to buy back bond in future
- Haircut

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Duration

31

Reverse Repo

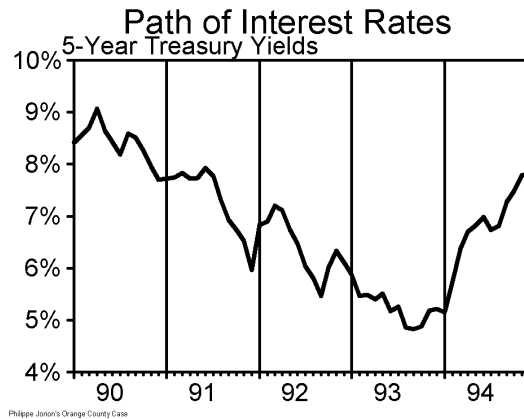


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32

Yield Movements



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Duration

33

Position (simplified)

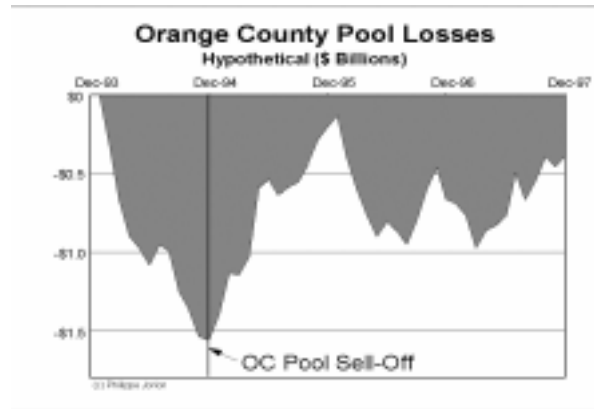
- \$7.8 B in contributed funds
- \$12.8 B borrowed funds: reverse repos
- \$20.6 B invested in 5 year T-notes
- Duration: 7.4
- Yields up by 2.6%

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34

Potential Losses



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35

Summary

- Applying duration
- convexity: intuition and formula
- Next Time
 - Swaps and convexity
 - Chapter 16, text
 - Factor Models for Bonds
 - Litterman and Scheinkman article
 - Kritzman on factor models

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36