Trading and Intro to TS

Goals:

- basic trading terminology
- Explain Law of One Price, and Arbitrage
- Calculate a replicating portfolio of bonds
- Explain relationship: Arbitrage and Replication
- Spot Rates
- Forward Rates

Organized Exchanges

- auction markets
- dealers
- Securities: stocks, futures contracts, options, bonds (somewhat)
- Examples: AMEX, NYSE, CBOE

OTC Market

- dealer market w/out centralized order flow
- NASDAQ: largest
- Scandal?
- Stocks, bonds, and some derivatives

Others

- 3rd Market: trading of listed securities away from exchange
- 4th market: institutions to institutions

Trading Costs

- Commission: paid to broker
- spread: cost of trading with dealer
  - bid: dealer buys
  - ask: dealer sells

Order Types

- basic idea:
- Market:
- Limit
- Stop loss

Margin Trading

- Maximum margin
  - currently 50%
  - set by Fed
- Maintenance margin
  - minimum equity margin can be
- margin call:

Example–Initial Conditions

Yahoo	\$70
50%	Initial Margin
40%	Maintenance Margin
1000	Shares purchased
Initial Position	
Stock	\$70,000
Borrowed	\$35,000
Equity	\$35,000

Maintenance Margin

- Stock Price falls to \$60 per share
- New position:
  - Stock \$60,000
  - Borrowed \$35,000
  - Equity \$25,000
- Margin = 25,000/60,000 = 41.67%
- Margin call: margin must drop to 40%. How much should price drop?

COUGARS Case

Questions

- Why have zero coupon bonds been sucessful?

- Relationship between COUGARS and coupon bonds?
- How much value was created in the COUGARS offering? Where did the value come from?

Arbitrage Opportunity

Strategy that:

- positive cash flow today
- no investment today
- no future obligations

Violates *Law of One Price*
example of violations?

Bonds

Are they really risk–free?

Bond Cash Flows

Pure discount bond, 3 year maturity with 1$ face value:

t:	0	1	2	3
Cash:		0	0	1

b_3 the bond price at $t = 0$.

t:	0	1	2	3
Cash:	$-b_3$	0	0	1

Short sale:

t:	0	1	2	3
Cash:	b_3	0	0	-1

Coupon bond with 3 year maturity, Face = 100\$, coupon rate = c%

t:	0	1	2	3
Cash:		c	c	$100 + c$

If the bond price were B_3 and you purchased:

t:	0	1	2	3
Cash:	$-B_3$	c	c	$100 + c$

Short Sale:

t:	0	1	2	3
Cash:	B_3	$-c$	$-c$	$-(100 + c)$

Real World

- t–bills
- t–bonds
- long–term bonds
- strips

Created by investment banks

How to Replicate

Can we make a 3 year coupon bond out of PDBs?

t:	0	1	2	3
coupon bond	$-B_3$	c	c	$100+c$
PD_1	$-b_1$	1	0	0
PD_2	$-b_2$	0	1	0
PD_3	$-b_3$	0	0	1

So if you buy:

- c units of PDB_1
- c units of PDB_2
- $100+c$ units of PDB_3

t:	0	1	2	3
Cash:	$-cb_1$	c	c	100+c
	$-cb_2$			
	$-(100+c)b_2$			

This gives the **same payout**
Cost of Replicant:

$$cb_1 + cb_2 + (100+c)b_3$$

Arbitrage and Replication

Suppose $B_3 < cb_1 + cb_2 + (100 + c)b_3$. To take advantage:

buy 1 coupon bond	$-B_3$	c	c	$100 + c$
sell c PDB_1	cb_1	$-c$	0	0
sell c PDB_2	cb_2	0	$-c$	0
sell $100 + c$ PDB_3	$(100 + c)b_3$	0	0	$-(100 + c)$
Net	$[cb_1 + cb_2$ $+(100 + c)b_3 - B_3]$	0	0	0

By assumption

$$cb_1 + cb_2 + (100 + c)b_3 - B_3 > 0$$

This meets our definition of an **ARBITRAGE**

Question: what to do if

$$cb_1 + cb_2 + (100 + c)b_3 < B_3$$

Punchline

- Law of One Price: same payoffs $\Longrightarrow$ same price
- Replication: PDB gives coupons, etc.
- Replication and Arbitrage

Question: No short sales?

Bond Prices and Interest Rates

b_1 price of 1 period PDB
Define $r_1 \equiv 1$ year interest rate

$$b_1 = \frac{1}{r_1} \Longrightarrow r_1 = \frac{1}{b_1}$$

$r_i \equiv i$ year interest rate

$$b_i = \frac{1}{r_i^i} \Longrightarrow r_i = \left(\frac{1}{b_i}\right)^{1/i}$$

Generally,$r_i \neq r_j \Longrightarrow$ **TERM STRUCTURE**

bond prices $\Longrightarrow$ term structure (spot interest rates)
Replication argument:

$$P = \sum_{i=1}^{N} b_i c f_i,$$

or,

$$P = \sum_{i=1}^{N} \frac{c f_i}{r_i^i},$$

Forward Rates

r_i today's rates on i period PDB's. Want to arrange borrowing/lending in the *future*.
e.g. meet today to arrange for advance borrowing 100$ in 1 yr. to be repaid in 2 yrs. **Forward Contract**

$${}_1f_2$$

Time:	0	1	2
Cash:	0	-100	$100\,{}_1f_2$

Rate fixed today: NO RISK

Forwards and Spots

Replication again!

1. buy a 2 year PDB. Get \$$100 r_2^2$ in two years.
2. buy a 1 year PDB and forward contract

Time	0	1	2
(1) 1 yr PDB	-100	$100r_1$	0
(2) forward	0	$-100r_1$	$100r_1\ {}_1f_2$
Net	-100	0	$100r_1\ {}_1f_2$
(1) 2 yr PDB	-100	0	$100r_2^2$

NO ARBITRAGE $\Longrightarrow r_1\ {}_1f_2 = r_2^2$,

$$ {}_1f_2 = \frac{r_2^2}{r_1} $$

r_i and b_i are related $b_i = \frac{1}{r_i^i}$

$$ \Longrightarrow\ {}_1f_2 = \frac{b_1}{b_2} $$

Do this for $i = 1, 2, ...$

$$ r_2^2\ {}_2f_3 = r_3^3 \Longrightarrow\ {}_2f_3 = \frac{r_3^3}{r_2^2} = \frac{b_2}{b_3} $$

$$ {}_nf_{n+1} = \frac{b_n}{b_{n+1}} = \frac{r_{n+1}^{n+1}}{r_n^n} $$

Spots and forwards are just interest rates. Can they be < 1?
PDB prices *decrease* as the maturity increases.

$$b_n \leq b_m \quad n > m$$

Example

$$b_1 = 0.81, \; b_2 = 0.9$$

To take advantage:

t	0	1	2
(1) buy PDB_1	-0.81	1	0
(2) sell PDB_2	0.90	0	-1
(3) matress from 1–2	0	-1	1
Net	0.11	0	0

Arbitrage!

Example: Replicating Portfolios, Spots, and Forwards
Coupon bond:
$M = 5yrs$, $c = 8\%$, $F = \$1,000$.

$$P = ?$$

Have the following \$ 10 PDB prices

$$b_1 = 9.346$$

$$b_2 = 8.654$$

$$b_3 = 7.939$$

$$b_4 = 7.217$$

$$b_5 = 6.561$$

Spot Rates

$$
\begin{aligned}
r_1 &= \left(\tfrac{10}{9.346}\right) &&= 1.07\\
r_2 &= \left(\tfrac{10}{8.654}\right)^{\frac{1}{2}} &&= 1.075\\
r_3 &= \left(\tfrac{10}{7.939}\right)^{\frac{1}{3}} &&= 1.08\\
r_4 &= \left(\tfrac{10}{7.217}\right)^{\frac{1}{4}} &&= 1.085\\
r_5 &= \left(\tfrac{10}{6.561}\right)^{\frac{1}{5}} &&= 1.088
\end{aligned}
$$

Forward Rates

$$ {}_1f_2 = \frac{(r_2)^2}{r_1} \qquad \text{Also,} \quad {}_1f_2 = \frac{b_1}{b_2} $$

$$ {}_1f_2 = \frac{(r_2)^2}{r_1} = \frac{(1.075)^2}{1.07} = 1.08 \qquad {}_1f_2 = \frac{b_1}{b_2} = \frac{9.346}{8.654} = 1.08 $$

$$ {}_2f_3 = \frac{(r_3)^3}{(r_2)^2} = \frac{(1.08)^3}{(1.075)^2} = 1.09 \qquad {}_2f_3 = \frac{b_2}{b_3} = \frac{8.654}{7.939} = 1.09 $$

$$ {}_3f_4 = \frac{(r_4)^4}{(r_3)^3} = \frac{(1.085)^4}{(1.08)^3} = 1.10 \qquad {}_3f_4 = \frac{b_3}{b_4} = \frac{7.939}{7.217} = 1.10 $$

$$ {}_4f_5 = \frac{(r_5)^5}{(r_4)^4} = \frac{(1.088)^5}{(1.085)^4} = 1.10 \qquad {}_4f_5 = \frac{b_4}{b_5} = \frac{7.217}{6.561} = 1.10 $$

Conclusions

- trading terminology
- Replication and arbitrage
- application to bonds and spot rates

Next Time

- When is a set of bonds arbitrage free
- Redemption yields