Investment Analysis — Homework # 1 due date: Tuesday, March 30, In Class

1. You see that two riskless 12% coupon government bonds are listed in the newspaper:

	Maturity	Coupon	Face Value	Price
Coupon Bond #1	1 year	12%	$100	$100
Coupon Bond #2	2 years	12%	$100	$97.67

   (a) Describe exactly how you could use these two coupon bonds to replicate one and two year pure discount bonds (face value $1).

   (b) What should be the prices of one and two year pure discount bonds (face value $1) if markets are perfect?

   (c) Based on your answer in part (b) above, is there an arbitrage opportunity in this situation? Why or why not?

   (d) You pay a visit to your bank manager, who says to you: "It is my belief that the riskless interest rate will stay constant at its gross rate of 1.12 p.a., and that it will not rise in future. Hence, if you want, the bank will **commit today** (i.e. at date 0) to lend you $100 in a year (i.e. at date 1) to be repaid two years from today (i.e. at date 2). The bank will commit to charge you the riskless rate, 1.12 p.a., on this loan. Thus, the amount you would have to repay at date 2 would be $112."
   Should you accept this proposal? How much is it worth to you **today**?

2. Two government bonds are being traded, both with maturity date 3 years from now. Bond 1 has a face value of $5,000 and pays an annual coupon of $400. Bond 2 has a face value of $1,000 and pays an annual coupon of $160. Bond 1 is trading at par. Bond 2 has a yield-to-maturity of 1.082 (i.e. 8.2%.).

   (a) What is the t=0 price of Bond 1? (Call it B_1).

   (b) What is the yield-to-maturity of Bond 1?

   (c) What is the t=0 price of Bond 2? (Call it B_2). Write the equation for calculating it.

   (d) Is it possible to make arbitrage profits using only these two bonds? If so, give an arbitrage strategy and demonstrate carefully that it generates arbitrage profits. If arbitrage is not possible, demonstrate carefully how you can be sure that arbitrage is impossible.

3. Two coupon bonds are being traded as follows:

Bond #	Price	Maturity	Face Value	Coupon Rate % p.a.
1	0.88	1 Year	$1	10%
2	1.07	2 Years	$1	10%

   (a) What is the yield to maturity on these bonds?

   (b) What are the prices of one and two year pure discount bonds, with faces values of $1?

   (c) Based on your answer above, is there an arbitrage opportunity in these bonds? Why or why not? If so, describe an arbitrage strategy.

4. You are given the following information: Two bonds are traded, both with maturity dates three years from now. Bond #1 has a face value of \$5,000 and pays an annual coupon of \$400. Bond #2 has an annual coupon rate of \$160, and a face value of \$1,000. Bond #1 is trading at par, and Bond #2 has a yield to maturity of 8.2%.
   (a) What is today's price of bond #1? Call this B_1.
   (b) What is the yield to maturity of bond #1?
   (c) What is today's price of bond #2? Call this B_2 and give the equation you used to calculate the price.
   (d) Is it possible to make arbitrage profits from these two bonds? If so, explain how, and if not, explain why you cannot.
   (e) Suppose there is a third bond, (#3) which matures *one* year from now, with a face value of \$1,000, a coupon rate of 0% and a yield to maturity of 8.5%. What is today's price of bond #3.
   (f) If you found an arbitrage with only bonds #1 and #2: is it possible to use bond #3 with bond #1 or #2 to come up with an arbitrage strategy that could not be matched by #1 and #2 only. If so, give the arbitrage strategy. If not, explain how you can be sure there is no arbitrage.
   (g) If you did not find an arbitrage with bonds #1 and #2 only: Can you use #1, #2 and #3 to make arbitrage profits. If so, present the arbitrage strategy. If not, explain carefully why you are sure that there is no arbitrage.
5. Cougars case. Solutions for this question should be done using Excel.
   (a) Are the COUGARS from the case priced correctly relative to the treasury securities given in the case? Calculate the cost of using COUGARS to replicate the coupon bonds, for the following coupon bonds:
      i. May, 1984, 15.75 coupon rate
      ii. November, 2003, 11-7/8 coupon rate

      Note: in for these bonds, coupons are paid *semi–annually*. Thus, a coupon bond with 16% coupon rate makes a coupon payment of $16/2 = 8$ per \$100 of face value on the coupon payment dates (every six months). Also, the prices of the coupon bonds given in the case are quoted in $\frac{1}{32}$ s. That is, if the price is quoted as 100.30, then you would pay 100 plus $\frac{30}{32}$ for the bond.
   (b) Calculate the implied spot rates from the coupon bonds given in the case and plot it relative to the implied spot rates from the COUGAR bond prices. Note: remember that the payments occur every 6 months on these bonds. You should annualize the implied spot rates when you calculate them. Also, you will need to make a decision about which bonds to use when you have multiple coupon bonds with the same maturity.
6. Suppose that the current price of a one period PDB is 0.95 and the current price of a 4 year PDB is 0.80.
   (a) What is the lowest possible two period spot rate of interest? That is, how much can the term structure slope down?
   (b) What is the highest possible two period spot rate of interest?
   (c) In general, can you say anything about how fast the term structure can slope down? I only want some intuition for this answer, you don't need to do any complicated algebra here.