

## Investment Analysis, Mini 4, 1999

### *Practice Questions for Final*

1. You are given the following default free bonds, with coupons paid once per year,

Bond	Coupon Rate	Face Value	Maturity	Price
A	5.00%	100	3 years	103.9
B	10.00%	100	3 years	117.8

- a. Is there an arbitrage opportunity in these bond prices? Why or why not?
  - b. You are trying to price a two year bond with a face value of 100 and a 5% yearly coupon rate. Given the above information, what is the highest price that you should be able to sell the bond for. What is the lowest price that you should be able to sell the bond for?
  - c. Suppose in addition to the above bond prices, you knew that the forward rate for forward contracts between year one and two was equal to 5.5%. Given this information and the bond prices given above, what is the price of a two year bond with face value 100 and 5% yearly coupons.
2. You are given the following information, coming from a mean-variance optimization model:

Portfolio	Mean	Standard Deviation	Weight in S&P 500	Weight in SML Portfolio	Weight in HML Portfolio
Global Minimum Variance	7.00%	5.00%	0.3	0.6	0.1
Efficient Portfolio	10.00%	10.00%	1	0	0
Riskfree rate	5.00%				

- a. Using the results of the above optimization, describe the optimal position for a client who wants an expected return of 8% without allowing for riskless borrowing or lending. What is the standard deviation of this position?
- b. How would you modify the above results if you allowed for riskless borrowing or lending? Describe what you would do, no computations are required here.
- c. You are a believer in the CAPM, and believe that the S&P 500 is the market portfolio. You would like an optimal portfolio with an expected return of 8%. Given the above information, describe the strategy that you would follow and the

- standard deviation that you would get from this strategy.
- d. Based on the above numbers, is the CAPM true for this economy? Why or why not.?
3. Your summer internship is to work for Short Term Capital Management. The firm is currently using a two factor model to price equities. The two factors consist of the return on the market portfolio and the return on a fixed income portfolio. The risk free rate is 1% per month,  $\lambda_{\text{market}} = 1\%$  per month, and  $\lambda_{\text{fixed-income}} = 2\%$  per month.
    - a. Your job is to look for mispriced equities, according to the firm's model. You think that you have found one. You think Merriwether Inc. has a beta with respect to the market of 0.5 and a beta with respect to the fixed income portfolio of 1.5, and Merriwether has a residual standard deviation of 1% per month. According to your analysis, the expected return on Merriwether Inc. is 6% per month. What does your pricing model say that the expected return for Merriwether should be?
    - b. Describe a strategy to take advantage of the mispricing that you found, using your factor model.
    - c. How would the answer to part b. change if you used the CAPM. That is, how would you exploit the mispricing in a CAPM world. You can still assume that Merriwether is mispriced, when you use the CAPM to do the pricing to answer this question.
  4. The current yield on a 2 year PDB is lower than the yield on the 30 year PDB. You believe that the spread between the yields on these bonds will widen over time.
    - a. Describe a trade to take advantage of your beliefs.
    - b. Now, you believe that a factor model best describes the yields of pure discount bonds. There is a one-factor model that describes the movements of the term structure. The loadings for the bonds are 0.75 on the 2 year PDB and 1.01 on the 30 year PDB. How does the trade you computed above now change?
    - c. For this part of the question, you can ignore the factor model from part c. You also have a 16 year PDB. Now, you do not want to trade on the spread of the rates. Instead, you also think that interest rate volatility is going to be very low over the next month. Structure a trade to take advantage of this, using the 2, 16 and 30 year PDBs. When will you make money with this trade? When will you lose money?
    - d. Restructure the trade in part c, using the factor model, with loadings of 0.75 on the 2 year, 1 on the 16 year and 1.01 on the 30 year.
  5. In this question, you are going to value a chooser option. This is an option that gives the holder the right to decide between a European put and a European call option at a particular point in time. The chooser option has a cost when entered into, and no money changes hands when the holder of the chooser receives the put or call. So, if the chooser option is purchased at time 0, and the chooser option expires at time 1, and the chooser is written on European puts and calls that expire at time 2 with exercise price X, then the time 1 cash flows from holding the chooser option are given by  $\max(P(S_1, X, 1), C(S_1, X, 1))$ . Here,  $P(S_1, X, 1)$  is the time 1 value of a put option at time 1, with 1 period to maturity, strike price X and underlying stock price  $S_1$ , with

$C(S_1, X, 1)$ , the call option value defined similarly.

- Show that a chooser option that matures in one year, on options that mature in 2 years be replicated with the underlying stock, bonds, and one and two year maturity calls. Hint: use put/call parity.
- Why might you want to purchase a chooser option?
- You are purchasing a chooser option on the stock whose dynamics are shown below. Here,  $u=1.25$ ,  $d=0.8$ . The spot interest rate is 10% per period, and won't change over the life of the contract. If you were purchasing a chooser option on two-period European options with exercise price 10, what would you pay for the chooser?

Stock Price Dynamics

