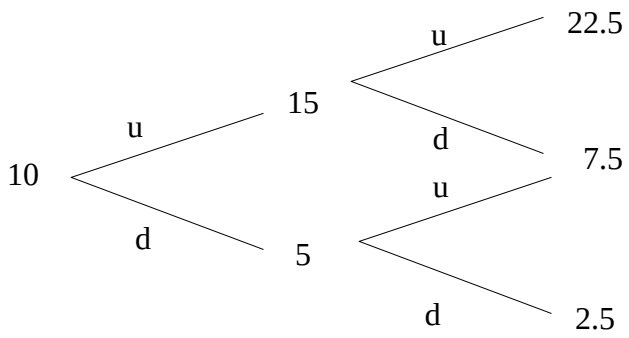
Objectives

- Valuing longer maturity options
  - how
  - hedging
  - implications of strategy
- Dynamic trading strategies
  - buy and hold, constant proportion
  - CPPI

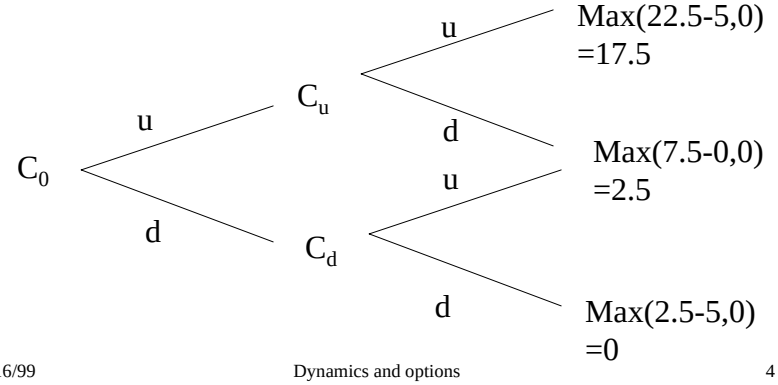
Valuing multi-period options

- Same basic idea: replicate final payoffs
- Example
  - S_0=10
  - u=1.5,d=0.5
  - r=1.1 per period
- Value call with 2 periods, X=5

Stock payoffs



Call option payoffs



Valuation

- Work backwards through tree.
- Replicate option value at each node
- Results
  - strategy of stock and bond with same payoffs as option
  - value of strategy = value of option

After initial ‘u’, or C_u

- # shares: α_u, # bonds: β_u bonds
- C_{uu}=17.5, C_{ud}=2.5, S_{uu}=22.5, S_{ud}=7.5, S_u=15

$$uu: \quad \alpha_u 22.5 + \beta_u 1.1 = 17.5$$
$$ud: \quad \alpha_u 7.5 + \beta_u 1.1 = 2.5$$
$$\rightarrow \alpha_u = 1, \beta_u = -5/1.1$$
$$Cost = 15 - 5/1.1 = 10.45 = C_u$$

After initial ‘d’

- # shares: α_d, # bonds: β_d bonds
- C_{du}=2.5, C_{uu}=0, S_{du}=7.5, S_{dd}=2.5, S_d=5

$$du: \quad \alpha_d 7.5 + \beta_d 1.1 = 2.5$$
$$dd: \quad \alpha_d 2.5 + \beta_d 1.1 = 0$$
$$\rightarrow \alpha_d = 0.5, \beta_d = -1.25/1.1$$
$$Cost = 0.5(5) - 1.25/1.1 = 1.36 = C_d$$

Initial value

- Replicate the value of the option in both the ‘u’ and ‘d’ states next period
- C_u=10.45,C_d=1.36, S_u=15,S_d=5,S_0=10

$$u: \quad \alpha 15 + \beta 1.1 = 10.45$$
$$d: \quad \alpha 5 + \beta 1.1 = 1.36$$
$$\rightarrow \alpha = 0.909, \beta = -3.2/1.1$$
$$Cost = 0.909(10) - 3.2/1.1 = 6.2 = C_0$$

Important points

- Probabilities of ‘u’ and ‘d’ don’t matter
- 2 securities, but 3 final outcomes for option
  - how can it work?
  - Need #securities greater than or equal to # states
- Dynamic trading

The positions

- Initial holdings
  - stock: 0.909, bonds -3.2/1.1
- Price rise
  - stock 1, bond -5/1.1
- Price fall
  - stock 0.5, bond -1.25/1.1
- trend chasing?

Key Points

- Replication over time
- Probabilities don't matter
- Dynamic trading can make many complicated payoffs
- Valuation technique
- Hedging technique

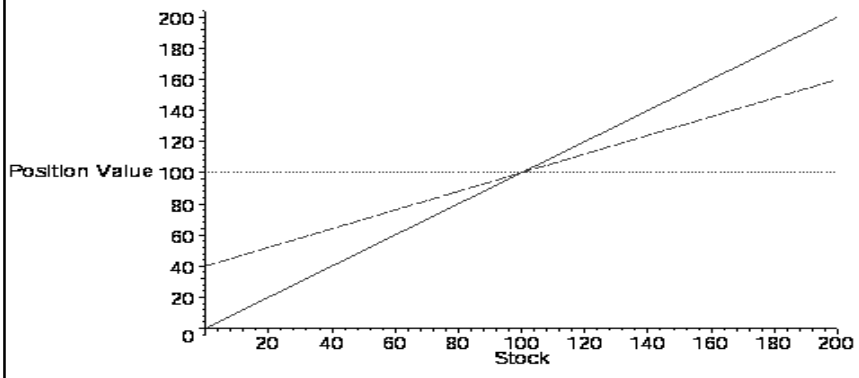
Black Scholes

- Same basic model:
- Let time between periods get 'small'
- Parameters
  - risk-free rate
  - stock volatility ('u' and 'd')
  - dividends
  - maturity of option

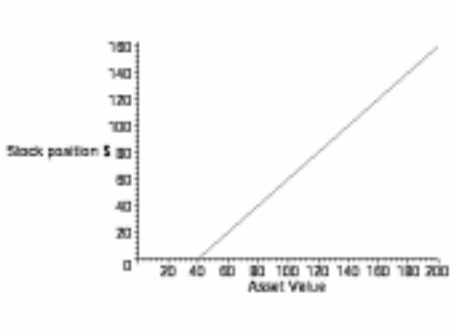
Dynamic trading strategies

- Sharpe and Perold
- Objectives:
  - understand what kind of dynamic strategies you could follow and why?
  - Mechanics of re-balancing

Buy and Hold Returns



Exposure Diagram 60/40 Buy-Hold



Constant Mix Strategies

- Invest constant % in risky asset
- Risk tolerance increases in wealth
- Dynamic Strategy
  - you need to trade to maintain the position

Example

- Start with $100
- $60 in stocks and $40 in Bills
- want to maintain constant weight

Case	Stock	Stock Value	Bill Value	Asset Value	% in Stocks
Initial	$100	$60	$40	$100	60
Market Drop	$90	$54	$40	$94	57.4
After re-balancing	$90	$56.4	$37.6	$94	60

Main Points

- Must rebalance to maintain constant weight
- Buy stocks when price falls and vice-versa when stock rises
- Issue of when to trade
  - typical rule: wait for % of price move
  - depends on volatility

Comparison: buy and hold and constant proportion

Case	Stock	Stock Value	Bill Value	Asset Value	% Stocks
Initial	100	60	40	100	60
Price drop	90	54	40	94	57.4
Rebalance	90	56.4	37.60	94	60
Rebound	100	62.67	37.60	100.27	62.5
Rebalance	100	60.16	40.11	100.27	60

Main Points

- Flat Volatile markets
  - constant proportion tends to do better
- Trends
  - better with `buy and hold'
- Trading costs?

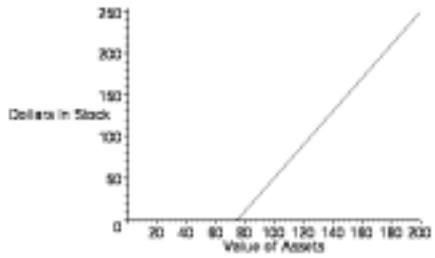
Constant Proportion Strategies

- Dollars in stock = m (assets - floor)
- m: multiplier
- m > 1: constant proportion portfolio insurance
- Who?

Example

- $100 wealth
- $75 floor
- multiplier of 2
- Initial cushion: 100-75=25
- Investment in stocks = 2(25)=50
- Preferences of investor?

Exposure Diagram



Example of Strategy

Case	Mkt Value Stock	Stock Value	Bill Value	Asset Value	Cushion	% in stocks
Initial	100	50	50	100	25	50
Market Drop	90	45	50	95	20	47.34
Rebalance	90	40	55	95	20	42.11
Drop	80	35.56	55	90.56	15.56	44.14
Rebalance	80	31.12	59.44	90.56	15.56	34.35
Rebound	100	37.33	59.44	96.78	21.77	38.57
Rebalance	100	43.54	53.24	96.78	21.77	44.98
Rebound	120	52.28	53.24	105.5	30.5	49.55
Rebalance	120	60.5	45.5	105.5	30.5	57.35

Main Points

- Sell stock in bear
- Buy stock in bull
- Relative to buy and hold
  - gain on downside
  - cost on upside
  - high enough, dominates buy and hold

What assets?

- Stocks and bonds only?

Underlying Assumptions

- When won't this work?

Payoff Shape

- Straight line: buy and hold
- Concave: buy stocks as they fall
  - constant mix
- Convex: buy stocks as they rise
  - CPPI and constant mix

Summary

- Dynamic replication of options
  - hedging
  - pricing
- Dynamic strategies
  - what kinds and why

Next Time

- Mean-variance analysis
  - basic assumptions and why?
- Introduction to implementation
- References
  - Text: Chapters 7-9 (brief review)
  - Kritzman on optimization (readings)