

## Objectives

- Application of factor model to portfolio optimization
- Explain what the CAPM means and what it doesn't
- Empirical evidence on CAPM
- Explain what APT is and why...

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## Estimating covariance matrix

- Lots of parameters in covariance matrix
  - with 10 assets, 55 parameters!
- Don't want too long data series
  - stationary issues
- Need to invert covariance matrix
- Solution
  - Factor models....

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## Factor Models and Portfolio Optimization

- Why
  - reduce number of parameters in var-cov matrix to estimate
- Plausible model

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## Example Single Factor Model

- For each asset,

$$\tilde{r}_{i,t} = E[\tilde{r}_{i,t}] + \beta_i \tilde{F}_t + \tilde{\varepsilon}_{i,t}$$

- Asset Variance

$$\sigma_i^2 = \beta_i^2 \sigma_F^2 + \sigma_{\varepsilon_i}^2$$

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## Variance-covariance matrix

- Calculate the covariances among assets

$$\begin{aligned} \text{Cov}(\tilde{r}_i, \tilde{r}_j) &= \text{Cov}(\beta_i \tilde{F} + \tilde{\varepsilon}_i, \beta_j \tilde{F} + \tilde{\varepsilon}_j) \\ &= \beta_i \beta_j \sigma_F^2 \end{aligned}$$

- With 10 assets
  - 10  $\beta$ 's
  - 10 residual variances and 1 factor variance
  - 21 parameters vs. 55 without factor assumption

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## Ways to get factors

- Factor Analysis: as in the bond case
- Economic Factors
  - market portfolio
  - interest rates or term structure
  - inflation
  - oil prices, etc.

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## Combining risky with riskless

- Find efficient portfolio with largest slope, or Sharpe ratio
- Can be done using solver in Excel

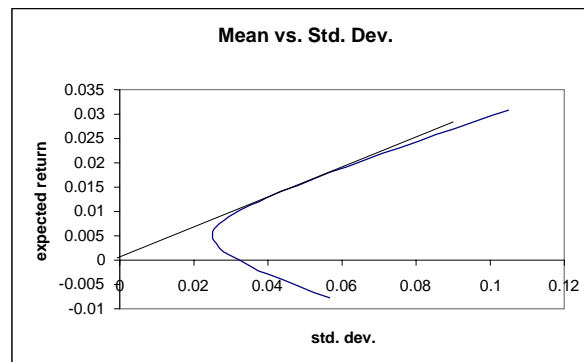
$$\max_{\tilde{r}_{port}} \frac{\left( E[\tilde{r}_{port}] - r_f \right)}{\sigma_{port}}$$

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## Optimal Portfolios



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## So far...

- Given mean and covariances can find optimal risky position
- Adding riskless
- Practical issues
  - sensitivity
  - where to get inputs

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## Equilibrium

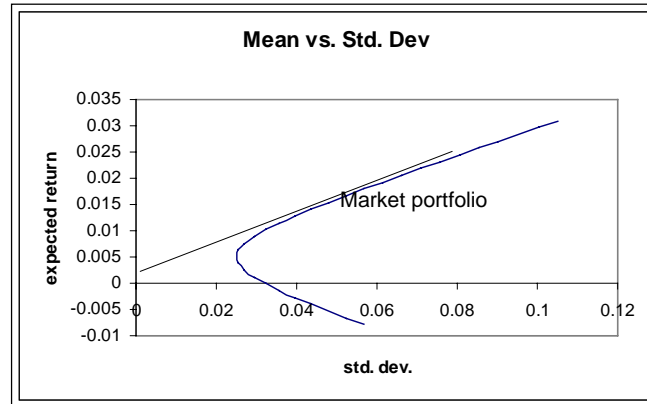
- Why worry about equilibrium?
  - Benchmark
  - Pricing results
- If we agree on picture
  - all pick same risky position
  - **market portfolio!**

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# CAPM



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## So What

- easy to figure out optimal strategy: combination of market and risk free
- For **all investments, efficient or not calculate beta relative to market**

$$E[\tilde{r}_i] = r_f + \beta_{i,market} (E[\tilde{r}_{market}] - r_f)$$

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## Discussion Questions

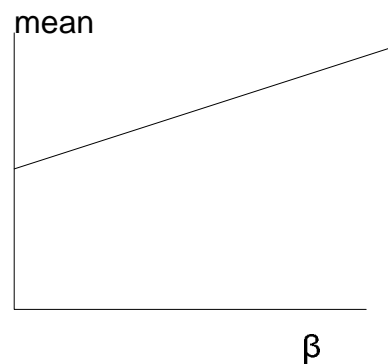
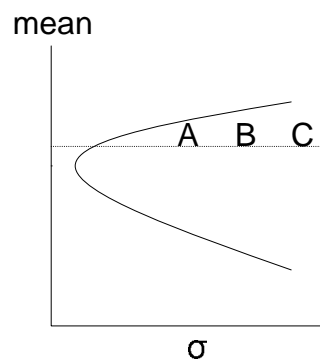
- You can't get higher return than market if you don't hold an efficient portfolio
- Stocks are riskier than bonds: all stocks should have higher expected return than risk free bonds
- You can't time the market

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## More Questions...



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## Yet More

- IBM has a beta of 1. Is it as risky as market?

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## Mispriced Assets in CAPM World

- Run the following regression
- CAPM: intercept ( $\alpha$ ) is suppose to be zero

$$\tilde{r}_i - r_f = \alpha_i + \beta_i(\tilde{r}_M - r_f) + \tilde{\varepsilon}_i$$

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## How to take advantage of non-zero $\alpha$ ...

- Suppose  $\alpha$  is positive
  - Asset has higher expected return than CAPM...
- Increase weights in that asset...
- Perhaps hedge with market.

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## Does $\beta$ explain everything?

- Anything?
- Fama and French (1992) results
  - sort stocks by  $\beta$  and price (size), B/M
  - cross-sectional regressions

$$E[\tilde{r}_i] = \gamma_0 + \gamma_{B/M}(B / M_i) + \gamma_{size} \log(ME) + \gamma_{\beta} \beta_i$$

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## Results

- The CAPM  $\beta$  explains almost nothing
- Size matters
  - small firm effect
- Book to market is important

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## Issues

- Time varying  $\beta$ 
  - seems to help a lot
- Data snooping
- Is this really CAPM?

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## Value Investing

- Take advantage of Book to Market Effect
- terrible over the last few years, but...

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## Momentum and Reversals

- Long and very short horizon reversals
  - 3 year reversals
  - one month reversals
- intermediate term momentum
  - 3 to 12 month holding periods

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## The APT

- CAPM
  - people only care about mean and variance
  - Intuition: only systematic risk should be priced
- APT
  - not based on investor preferences but on ‘factor model’ for returns

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## APT Model

- Common factors that determine co-movements of returns
- systematic and diversifiable risk
- Intuition of model
  - only systematic risk is rewarded...

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## Factor Model

$$\tilde{r}_i = E[\tilde{r}_i] + \beta_i^1 \tilde{F}^1 + \beta_i^2 \tilde{F}^2 + \dots + \beta_i^K \tilde{F}^K + \tilde{\varepsilon}_i$$

- F: factor
- $\beta$ : factor loading
- $\varepsilon$ : diversifiable risk

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## Pricing

- Expected returns proportional to factor loadings
- Intuition: only systematic risk should command higher expected returns

$$E[\tilde{r}_i] = r_{free} + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + \dots + \lambda_K \beta_i^K$$

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## Example

- 3 factors
- Stock  $i$  has
  - $\beta_i^1=1.1$ ,  $\beta_i^2=0.5$ ,  $\beta_i^3=2$
  - $\lambda_1=10\%$ ,  $\lambda_2=10\%$ ,  $\lambda_3=1\%$ , Riskfree rate =5%
- Expected return on stock?

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## Summary

- CAPM
  - what it means in words
  - applications
- APT
  - intuition

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