Bioimage Informatics

Lecture 20, Spring 2012

Basic Diffusion Theory

Biological Applications (II)

Experimental and Computational Analysis of

Spindle Microtubule Flux



Center for Computational Biology
Carnegie Mellon

Outline

- Review: computational analysis of axonal transport
- Basic diffusion theory
- Computational analysis of Spindle Microtubule Flux

- Review: computational analysis of axonal transport
- Basic diffusion theory
- Computational analysis of spindle microtubule flux

Axonal Cargo Transport (I)



- Axonal transport is critical to survival and function of neurons.
- Axonal transport provides a powerful model of intracellular transport.

A Drosophila Model of Alzheimer's Disease

- Two pathological hallmarks of AD: Aβ plaques & tau tangles
- Control:
 - SG26.1 GAL4/+; UAS-APPYFP/+ ← transport is driven by kinesin-1
- Mutants:

```
SG26.1 GAL4/+; UAS-APPYFP/+; UAS-wt hTau/+
SG26.1 GAL4/+; UAS-APPYFP/+; UAS-R406W hTau/+
```



Wittmann et al, Science, 2001

The questions:

1) What are the differences between normal and degenerative neurons in their axonal transport behaviors?

2) What causes transport defects in degenerative neurons?

Tracking Vesicle Movement Using Computer Vision Techniques (I)



Some General Comments (I)

- To select or build effective visualization tools is very important to the development of biological image analysis algorithms.
- It is critical to recognize and prevent potential information loss in the analysis work flow.
- Because of the small number of features, it is feasible to use algorithms with high computational complexity.



Some General Comments (II)

- A more comprehensive description of the work flow of particle tracking.
- What problems do you see in this picture?



Meijering et al, *<u>Tracking in Molecular Bioimaging</u>*, IEEE Signal Processing Magazine, 2006.

Tracking Vesicle Movement Using Computer Vision Techniques (II)





APP Vesicle Transport and its Impairment is Region-Specific



How can we make sure that information loss is minimized?



Questions for identifying potential information loss

1) Does each vesicle change its velocity over time? If so, how?

2) How are the vesicles spatially distributed?

Tau Overexpression Differentially Affects Axonal Transport





Axon Swelling and Vesicles Accumulation



10 um



What can we learn from this? 1) How the images should be analyzed is strongly dependent on the biological questions to be addressed. 2) It is important to identify research questions from applications.

Challenge: To Infer Mechanisms from Behaviors



- Review: computational analysis of axonal transport
- Basic diffusion theory
- Computational analysis of spindle microtubule flux

Thermal Movement of a Free Molecule

• The average kinetic energy of a particle of mass *m* and velocity v_x is

$$\left\langle \frac{1}{2} m v_x^2 \right\rangle = \frac{kT}{2} \qquad \begin{array}{l} \text{Boltzmann constant} = 1.381 \times 10^{-23} \text{ J/K} \\ 1 \text{ Joule} = 1 \text{ N} \cdot \text{m} \qquad t_{\text{K}} = t_{\text{C}} + 273.15 \end{array}$$

where k is Boltzmann's constant and T is absolute temperature (Einstein 1905).

 Molecular mass of GFP is 27 kDa. One atomic mass unit (Da) is 1.6606×10⁻²⁴g. So the mass of one GFP molecule is 4.4836×10⁻²⁰g.

At 27 degree C, kT is $4.1451 \times 10^{-14} \text{g} \cdot \text{cm}^2/\text{sec}^2$.

$$\sqrt{\langle v_x^2 \rangle} = \sqrt{\frac{kT}{m}} = 961.51 \text{ cm/sec}$$

1D Random Walk in Solution (I)

- Assumptions: consider an ensemble of N particles,
 - (1) A particle *i* has equal probabilities to walk to the left and to the right.
 - (2) Particle movement at consecutive time points are independent.
 - (3) Movement of different particles are independent.
 - (4) Each particle moves at a average step size of $\delta = v_x \cdot \tau$

$$x_i(n) = x_i(n-1) \pm \delta$$

$$-3\delta -2\delta -\delta 0 +\delta +2\delta +3\delta$$
$$\left\langle x(n)\right\rangle = \frac{1}{N} \sum_{i=1}^{N} x_i(n) = \frac{1}{N} \sum_{i=1}^{N} \left[x_i(n-1) \pm \delta \right]$$
$$= \frac{1}{N} \sum_{i=1}^{N} x_i(n-1) = \left\langle x(n-1) \right\rangle$$

• Property 1: The mean position of an ensemble of particles undergoing random walk remains unchanged.

1D Random Walk in Solution (II)

• Property 2: The mean square displacement of a particle undergoing random walk increases linearly w.r.t. time.

$$\left\langle x^{2}(n) \right\rangle = \frac{1}{N} \sum_{i=1}^{N} x_{i}^{2}(n) = \frac{1}{N} \sum_{i=1}^{N} \left[x_{i}^{2}(n-1) \pm 2\delta x_{i}(n-1) + \delta^{2} \right]$$
$$= \left\langle x^{2}(n-1) \right\rangle + \delta^{2}$$

$$\langle x^2(n) \rangle = n\delta^2 = \frac{t}{\tau}\delta^2 = 2Dt$$

Howard Berg, *Random walks in biology*, Princeton University Press, 1993

Application of the Microscopic Theory (I)

Object	Distance diffused			
	1 μm	100 µm	1 cm	1 m
K ⁺	0.25ms	2.5s	2.5×10 ⁴ s (7 hrs)	2.5×10 ⁸ s (8 yrs)
Protein	5ms	50s	5.0×10 ⁵ s (6 days)	5.0×10 ⁹ s (150 yrs)
Organelle	1s	10 ⁴ s (3 hrs)	10 ⁸ s (3 yrs)	10 ¹² s (31710 yers)

K+: Radius = 0.1nm, viscosity = 1mPa·s⁻¹; T = 25°C; D=2000 μ m²/sec Protein: Radius = 3nm, viscosity = 0.6915mPa·s⁻¹; T = 37; D = 100 μ m²/sec Organelle: Radis = 500nm, viscosity = 0.8904mPa·s⁻¹; T = 25°C; D = 0.5 μ m²/sec

Jonathon Howard, *Mechanics of motor* proteins and the cytoskeleton, Sinauer, 2001

Application of the Microscopic Theory (II)



H. Qian, M. P. Sheetz, E. L. Elson, *Single particle tracking: analysis of diffusion and flow in two-dimensional systems*, Biophysical Journal, 60(4):910-921, 1991.

- Review: computational analysis of axonal transport
- Basic diffusion theory
- Computational analysis of spindle microtubule flux

Overview of Cell Cycle



Dynamic Microtubules in the Mitotic Spindle





Green: microtubule Red: kinetochore



Confirmation of Poleward Flow of Spindle Microtubules



5 μm ——

00:00

Cameron et al, JCB, 173:173-179,2006

Fluorescent Speckle Microscopy (FSM)



FSM of Dynamic Spindle Architecture



Fluorescent speckle microscopy

Quantitative Mapping of Spatial-Temporal Spindle Dynamics



Yang et al., J. Cell Biol., 182:631-639, 2008

Regional Variations of Microtubule Flux



Yang et al., J. Cell Biol., 182:631-639, 2008

Questions?