Bioimage Informatics

Lecture 17, Spring 2012

Bioimage Data Analysis (V)

Single Particle Tracking (part 2)
Outline

• Basic concept of single particle tracking
• Graph assignment based single particle tracking
• Application I: fluorescence speckle microscopy
• Basic concept of single particle tracking
  • Graph assignment based single particle tracking
  • Application I: fluorescence speckle microscopy
Feature Detection Demo
Feature Tracking Demo
Definition of Particle Tracking (I)

• The goal is to fully recover the trajectory of each point feature, i.e. to determine the position of each point in each frame in which it exists.

For particle $k$, its trajectory is the sequence of its position coordinates in each frame within its total lifetime of $N$, i.e.

$$(x_k^1, y_k^1), (x_k^2, y_k^2), \ldots (x_k^N, y_k^N)$$
Definition of Particle Tracking (II)

• Different cases
  - Constant number of features
  - Feature appearance
  - Feature disappearance

• Cases of feature appearance & disappearance
  - Moving in or out of field of view
  - Moving in or out of the focal plane
  - Assembly/disassembly
  - Feature merging/splitting
Example: Particle Tracking (I)

- Frame $i-1$
- Frame $i$
- Frame $i+1$
Example: Particle Tracking (II)

- Frame $i$-1
- Frame $i$
- Frame $i$+1
Example of Particle Tracking (III)

- Frame $i-1$
- Frame $i$
- Frame $i+1$
Discussion: Different Tracking Strategies

• **Strategy I**: If the point correspondence between each pair of frames can be determined, the point correspondence over the entire image sequence is defined.
  - Advantages: relatively simple to implement
  - Disadvantages: a greedy approach, inadequate information to make a decision.

• **Strategy II**: to establish point correspondence based on information from multiple frames.
  - Advantages: decision making is more reliable.
  - Disadvantages: computationally intractable in most cases.

• **Solution**: to find a solution in between strategy I and II
An Example of Conflict Resolution

• Basic concept of single particle tracking

• Graph assignment based single particle tracking

• Application I: fluorescence speckle microscopy
Particle Tracking Based on Global Linear Assignment (I)

- An optimization strategy is required to resolve conflicts between competing assignments.

- Selection of assignment weight will critically influence outcomes.

Particle Tracking Based on Global Linear Assignment (II)

- Formulation of the tracking problem as a bipartite graph assignment

\[
\min \sum_{i \in G_k} \sum_{j \in G_{k+1}} a^k(i, j) w^k(i, j)
\]

st. \( \sum_i a(i, j) = 1 \quad \sum_j a(i, j) = 1 \quad a(i, j) \in \{0, 1\} \)

- There are efficient numerical algorithms to solve large scale assignment problems.

- Why not use a tripartite graph?
  - Optimal assignment of tripartite graph is NP-complete.
  - Difficult to resolve conflicts between two tripartite assignments.
Commonly Used Assignment Weight Definitions

• Distance → Nearest neighbor

\[ c^k(i, j) = \|x_{j}^{k+1} - x_i^k\| \]

• Smooth motion → Smooth motion

\[
\begin{align*}
  c^k(i, j) &= w_1 \left[ 1 - \frac{(x_i^k - x_{i-1}^k)(x_{j+1}^k - x_i^k)}{\|x_i^k - x_{i-1}^k\| \|x_{j+1}^k - x_i^k\|} \right] + w_2 \left[ 1 - 2 \sqrt{\frac{\|x_i^k - x_{i-1}^k\| + \|x_{j+1}^k - x_i^k\|}{\|x_i^k - x_{i-1}^k\| + \|x_{j+1}^k - x_i^k\|}} \right]
\end{align*}
\]

• Mahalanobis distance, where the prediction comes from typically a Kalman filter

\[
\begin{align*}
  c^k(i, j) &= (x_i^k - \hat{x}_i^k)^T S(x_i^k)^{-1} (x_i^k - \hat{x}_i^k)
\end{align*}
\]
How to Handle Particle Appearance & Disappearance

• Track appearance and disappearance are handled by introducing virtual points.

G. Yang, A. Matov, G. Danuser, Reliable tracking of large scale dense antiparallel particle motion for fluorescence live cell imaging, *IEEE CVPR, 2005*
References on Linear Assignment


(Downloadable from http://ccdl.compbio.cmu.edu/BME42_731/Burkard_LAP_review.pdf).
An Overview of Axonal Transport

• Axonal transport is critical to survival and function of neurons.
• Axonal transport is a powerful model of intracellular transport.
• Axonal transport may be a good model to study spatiotemporal cell signaling.
• Many mitotic motors also drive axonal transport.
Molecular Motor Machinery of Axonal Transport

Potential Mechanisms of Axonal Transport Defects
MHT-Based Particle Tracking (I)

- “MHT is a deferred decision logic in which alternative data association hypotheses are formed whenever observation-to-track conflict situations occur.”

- Multiple competing hypotheses are represented in a tree structure.

![Diagram of MHT tree structure with N-scan pruning](image-url)
MHT-Based Particle Tracking (II)

- The tree structure provides a flexible way to handle feature appearance/disappearance and merging/splitting.

Fig. 5. Formation of hypotheses from tracks in families.
MHT-Based Particle Tracking (III)

• **Advantages**
  - An effective framework to incorporate multi-frame information.
  - A natural way to handle feature appearance and disappearance.
  - A natural way to handle feature merging/splitting.

• **Disadvantages**
  - Combinatorial explosion if the tree is not pruned.
  - Many variations in implementation.
  - High computation and memory cost.

• Overall a very important approach, especially when the number of features to be tracked is small.
Methods to Handling Merging & Splitting

• **Tracklet-based approaches**

• **Graph-based approaches**
• Basic concept of single particle tracking

• Graph assignment based single particle tracking

• Application I: fluorescence speckle microscopy
Overview of Cell Cycle
Dynamic Microtubules in the Mitotic Spindle

Alberts et al., MBoC5
Confirmation of Poleward Flow of Spindle Microtubules

Cameron et al, JCB, 173:173-179, 2006
Fluorescent Speckle Microscopy (FSM)
FSM of Dynamic Spindle Architecture

Fluorescent speckle microscopy
Quantitative Mapping of Spatial-Temporal Spindle Dynamics

Regional Variations of Microtubule Flux

Questions?
Movement of a Free Molecule (I)

- The average kinetic energy of a particle of mass $m$ and velocity $v_x$ is

$$\left\langle \frac{1}{2} m v_x^2 \right\rangle = \frac{k T}{2}$$

where $k$ is Boltzmann's constant and $T$ is absolute temperature (Einstein 1905).

- Principle of equipartition of energy

$$\left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3 \cdot k T}{2}$$

Boltzmann constant $= 1.381 \times 10^{-23}$ J/K

1 Joule = 1 N·m

$t_K = t_C + 273.15$

Howard Berg, Random walks in biology, Princeton University Press, 1993
Movement of a Free Molecule (II)

• Molecular mass of GFP is 27 kDa. One atomic mass unit (Da) is $1.6606 \times 10^{-24}$ g. So the mass of one GFP molecule is $4.4836 \times 10^{-20}$ g.

At 27 degree C, $kT$ is $4.1451 \times 10^{-14}$ g cm$^2$/sec$^2$.

$$\sqrt{\langle v_x^2 \rangle} = \sqrt{\frac{kT}{m}} = 961.51 \text{ cm/sec}$$

Howard Berg, Random walks in biology, Princeton University Press, 1993
1D Random Walk in Solution (I)

- **Assumptions:** *consider an ensemble of N particles*,

  (1) A particle $i$ has equal probabilities to walk to the left and to the right.
  (2) Particle movement at consecutive time points are independent.
  (3) Movement of different particles are independent.
  (4) Each particle moves at a average step size of $\delta = v_x \cdot \tau$

\[
x_i(n) = x_i(n-1) \pm \delta
\]

\[
\langle x(n) \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i(n) = \frac{1}{N} \sum_{i=1}^{N} [x_i(n-1) \pm \delta]
\]

\[
= \frac{1}{N} \sum_{i=1}^{N} x_i(n-1) = \langle x(n-1) \rangle
\]

- **Property 1:** The mean position of an ensemble of particles undergoing random walk remains unchanged.

Howard Berg, Random walks in biology, Princeton University Press, 1993
1D Random Walk in Solution (II)

- Property 2: The mean square displacement of a particle undergoing random walk increases linearly w.r.t. time.

\[
\left\langle x^2(n) \right\rangle = \frac{1}{N} \sum_{i=1}^{N} x_i^2(n) = \frac{1}{N} \sum_{i=1}^{N} \left[ x_i^2(n-1) \pm 2\delta x_i(n-1) + \delta^2 \right] \\
= \left\langle x^2(n-1) \right\rangle + \delta^2
\]

\[
\left\langle x^2(n) \right\rangle = n\delta^2 = \frac{t}{\tau} \delta^2 = 2Dt \\
\left\langle r^2(n) \right\rangle = \left\langle x^2(n) + y^2(n) \right\rangle = 4Dt \\
\left\langle r^2(n) \right\rangle = \left\langle x^2(n) + y^2(n) + z^2(n) \right\rangle = 6Dt
\]

Application of the Microscopic Theory (I)

<table>
<thead>
<tr>
<th>Object</th>
<th>Distance diffused</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 μm</td>
</tr>
<tr>
<td>K⁺</td>
<td>0.25ms</td>
</tr>
<tr>
<td>Protein</td>
<td>5ms</td>
</tr>
<tr>
<td>Organelle</td>
<td>1s</td>
</tr>
</tbody>
</table>

K⁺: Radius = 0.1nm, viscosity = 1mPa·s⁻¹; T = 25°C; D=2000 μm²/sec
Protein: Radius = 3nm, viscosity = 0.6915mPa·s⁻¹; T = 37; D = 100 μm²/sec
Organelle: Radis = 500nm, viscosity = 0.8904mPa·s⁻¹; T = 25°C; D = 0.5 μm²/sec

Application of the Microscopic Theory (II)

Application of the Microscopic Theory (III)

• Calculation of diffusion coefficient (Einstein-Stokes equation)
  - diffusion of spherical particles through liquid in which viscous force dominates

\[ D = \frac{kT}{6\pi\eta r} \]

• Boltzmann constant: \( k = 1.381 \times 10^{-23} \text{J/k} = 1.381 \times 10^{-17} \text{N}\cdot\mu\text{m/k} \)
• Absolute temperature: \( T = 273.15 + 25 \)
• Viscosity: \( \eta = 0.8904 \text{mPa}\cdot\text{s} = 0.8904 \times 10^{-3} \times 10^{-12} \text{N}\cdot\mu\text{m}^{-2}\cdot\text{s} \)
• Sphere radius: \( r = 500 \text{nm} = 0.5 \mu\text{m} \)
• Calculated diffusion coefficient: \( D = 0.5 \mu\text{m}^2/\text{s} \)