Bioimage Informatics

Lecture 16, Spring 2012

Bioimage Data Analysis (IV)

Image Segmentation (part 5)

Bioimage Data Analysis (V)

Single Particle Tracking (part 1)

Outline

- Review: perceptual organization
- Graph-cut based image segmentation
- Active contour based image segmentation

- Basic concept of single particle tracking
- Graph assignment based single particle tracking

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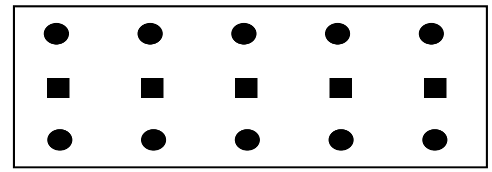
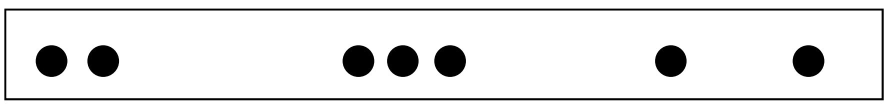
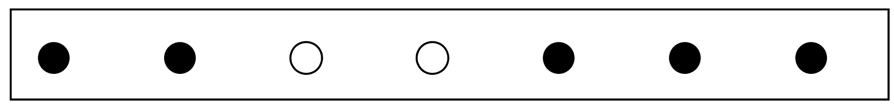
Perceptual Organization in Human Vision (I)

- The Gestalt school of psychology originated in 1920s-1930s in recognition of the role of perceptual organization in human vision.
- Gestalt: a German word meaning "form" or "whole"
- Some basic principles of perceptual organization

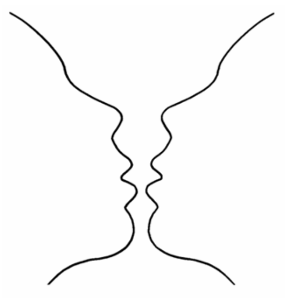




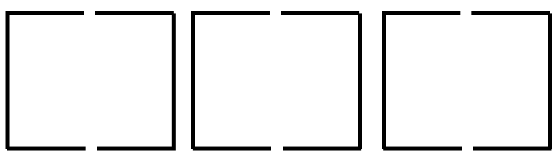
proximity



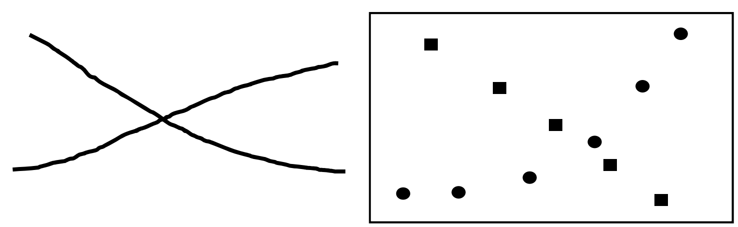
similarity



symmetry



closure

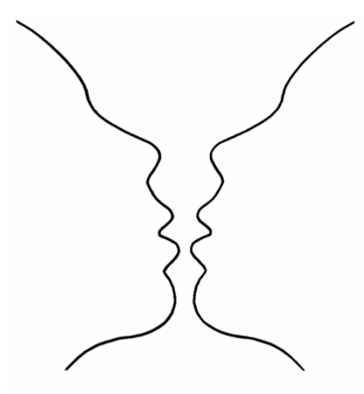


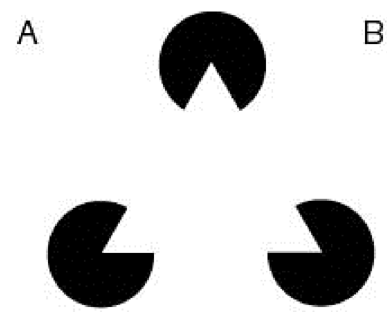
continuity

Perceptual Organization in Human Vision (III)

- Some fundamental questions

    - Example: The law of closure





What to compute?
How to compute?

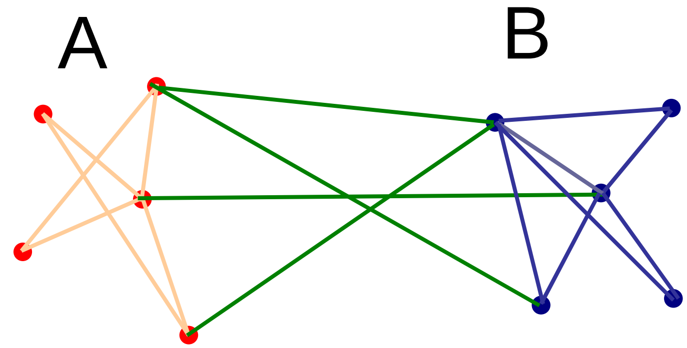
- Review: perceptual organization
- Graph-cut based image segmentation
- Active contour based image segmentation
- Basic concept of single particle tracking
- Graph assignment based single particle tracking

Basic Concept of Graph Cuts (I)

- A graph G = (V, E) can be partitioned into two disjoint sets A, B

$$A \bigcup B = V \qquad A \bigcap B = \varnothing$$

- Each vertex represents a pixel within the image.
- The weight of the edge connecting two vertices represents their similarity.

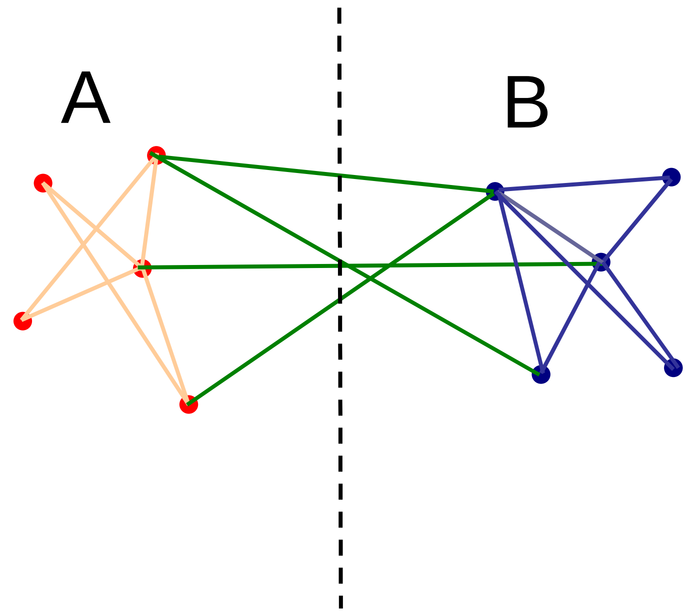


Shi & Malik, PAMI, 22:888-905, 2000

Basic Concept of Graph Cuts (II)

- A graph cut is defined as

$$cut(A,B) = \sum_{u \in A, v \in B} w(u,v)$$



- The goal is to find a partition that minimizes the cut.

- But there is a catch: minimum cut favors small sets of isolated nodes.

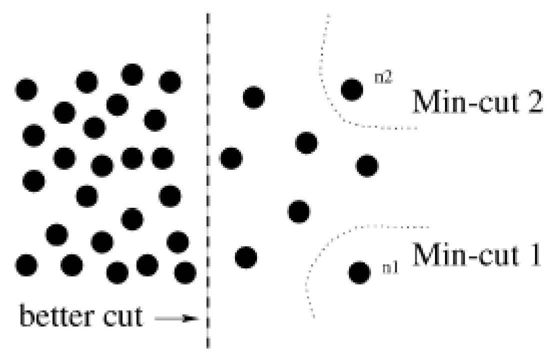


Fig. 1. A case where minimum cut gives a bad partition.

Formulation of Normalized Cut (I)

- Definition of normalized cut

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

where $assoc(A,V) = \sum_{u \in A, t \in V} w(u,t) = assoc(A,A) + cut(A,B)$

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,A) + cut(A,B)} + \frac{cut(A,B)}{assoc(B,B) + cut(A,B)}$$

Formulation of Normalized Cut (II)

- Definition of normalized association

$$Nassoc(A,B) = \frac{assoc(A,A)}{assoc(A,V)} + \frac{assoc(B,B)}{assoc(B,V)}$$

$$= \frac{assoc(A,V) - cut(A,B)}{assoc(A,V)} + \frac{assoc(B,V) - cut(A,B)}{assoc(B,V)}$$

$$= 2 - \left(\frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)} \right)$$

$$\boxed{Nassoc(A,B) = 2 - Ncut(A,B)}$$

- Minimization of normalized cut is equivalent to maximization of normalized association.

Solution of Normalized Cut (I)

- Matrix formulation

$$Ncut(A,B) = \frac{\sum_{(x_i>0, x_j<0)} -w_{ij} x_i x_j}{\sum_{x_i>0} d_i} + \frac{\sum_{(x_i<0, x_j>0)} -w_{ij} x_i x_j}{\sum_{x_i<0} d_i}$$

$$d(i) = \sum_j w(i,j)$$

- Reformulate the problem into a matrix form

$$D = \begin{bmatrix} d_1 & 0 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & d_{N-1} & 0 \\ 0 & 0 & 0 & 0 & d_N \end{bmatrix} \text{ where } d_i = \sum_j w(i,j) \quad W(i,j) = w(i,j)$$

Solution of Normalized Cut (II)

- Solution of the normalized cut can be transformed into the minimization of the Rayleigh quotient.

$$min_x \; Ncut(x) = min_y \frac{y^T (D-W) y}{y^T D y}$$

$$\text{where} \quad y = (1+x) - b(1-x), \qquad b = \frac{k}{1-k}, \quad k = \frac{\sum_{x_i > 0} d_i}{\sum d_i}, \quad d_i = \sum_j w(i,j)$$

$$s.t. \quad y_i \in \{1, -b\} \qquad y^T D 1 = 0$$

- Exact solution of normalized cut is NP-complete.

Solution of Normalized Cut (III)

- A key relaxation is to allow y to take on continuous real values. Now y can be determined as the solution of the following generalized eigenvalue problem.

$$\left(D-W\right)y=\lambda Dy$$

- After a further transformation, y is the solution of the following equation

$$D^{-\frac{1}{2}}\left(D-W\right)D^{-\frac{1}{2}}z=\lambda z \text{ where } y=D^{-\frac{1}{2}}z$$

$$\min_x Ncut\left(x\right)=\min_y \frac{y^T\left(D-W\right)y}{y^T Dy}$$

$$\Updownarrow$$

$$\left(D-W\right)y=\lambda Dy$$

$$\Updownarrow$$

$$D^{\frac{1}{2}}\left(D-W\right)D^{-\frac{1}{2}}z=\lambda z \qquad z=D^{\frac{1}{2}}y$$

Summary of the Solution Procedure

- Step 0: Compute *D, W*

- Step 1: Solve the eigenvector of the following equation

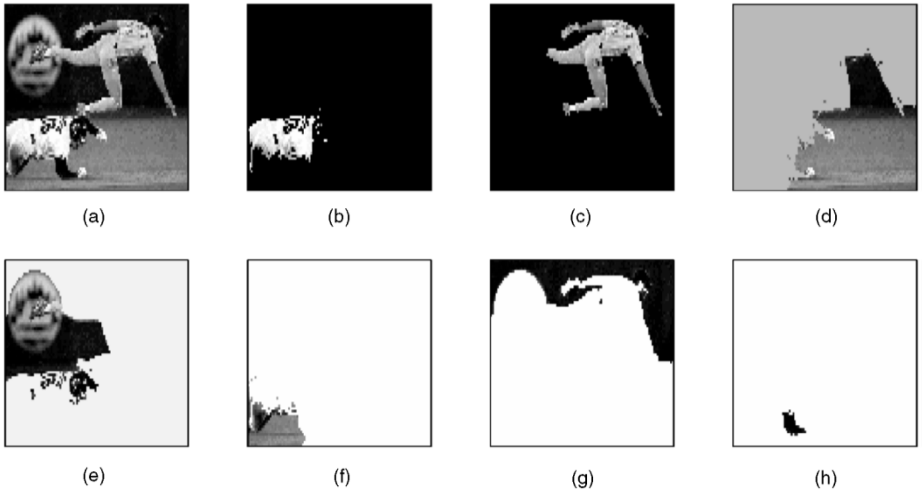
$$D^{-\frac{1}{2}}\left(D-W\right)D^{-\frac{1}{2}}z=\lambda z$$

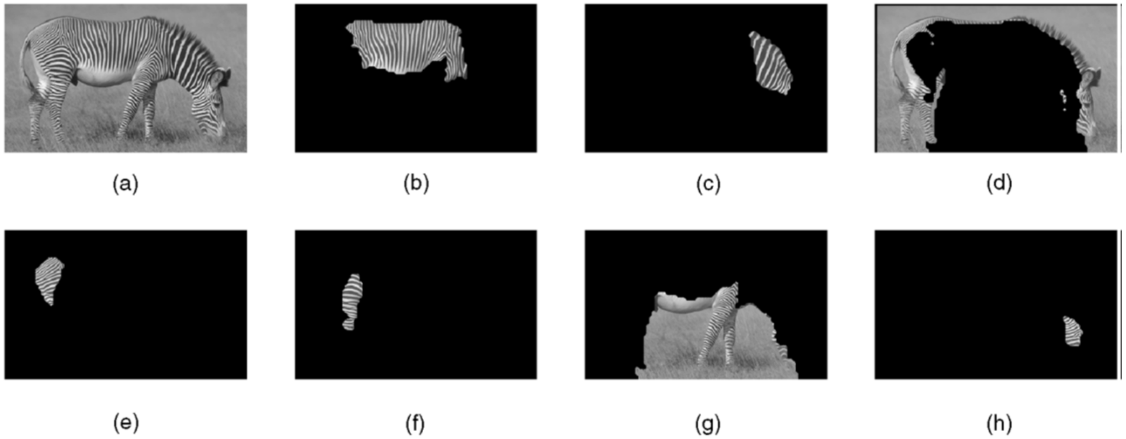
- Step 2: Take the eigenvector corresponding to the second smallest eigenvalue and calculated

$$y=D^{-\frac{1}{2}}z$$

- Step 3: Partition y
  - By taking zero or the median as the splitting point
  - Search for the splitting point that minimizes *Ncut*

Results





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Basic Idea of Active Contour (I)

- The basic idea is to start with an initial contour and iteratively update through an energy minimization process such that the contour will actively converge to the boundary of the object of interest.

$$E = E_{\text{Internal}} + E_{\text{external}} + E_{\text{interaction}}$$

- Internal energy: to control shape of the contour
- External energy: to control convergence
- Interaction energy: to added user-defined constraints

Kass, Witkin, Terzopoulos, *Snakes: active contour models*, IJCV, 321-331, 1988.

Basic Idea of Active Contour (II)

- Definition of internal energy

$$E_{\text{internal}} = \int_0^1 \alpha(s)\left|f'(s)\right|^2 + \beta(s)\left|f''(s)\right|^2$$

$$= \sum_i \alpha(i)\left\| f(i+1) - f(i)\right\|^2 / h^2 + \beta(i)\left\| f(i+1) - 2f(i) + f(i-1)\right\|^2 / h^4$$

- Definition of external energy

$$E_{\text{external}} = w_{\text{line}} E_{\text{line}} + w_{\text{edge}} E_{\text{edge}} + w_{\text{terminal}} E_{\text{terminal}}$$

$$E_{\text{edge}} = \sum_i -\left\|\nabla I\left(f(i)\right)\right\|^2 = \sum_i -\left\|\nabla I\left(x(i), y(i)\right)\right\|^2$$

Comments on Active Contour

- Strengths and weaknesses of SNAKES

- Level set methods

  S. Osher, N. Paragios, Geometrical Level Set Methods in Imaging, Vision, and Graphics, Springer, 2003.

- Two simple demonstrations

  - An implementation of the classic approach

    http://www.markschulze.net/snakes/

  - An implementation of the GVF approach

    http://www.iacl.ece.jhu.edu/static/gvf/

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Definition of Particle Tracking (I)

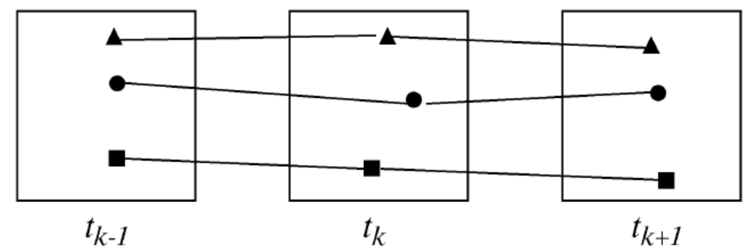
- The goal is to fully recover the trajectory of each point feature, i.e. to determine the position of each point in each frame in which it exists.

For particle k, its trajectory is the sequence of its position coordinates in each frame within its total lifetime of N, i.e.

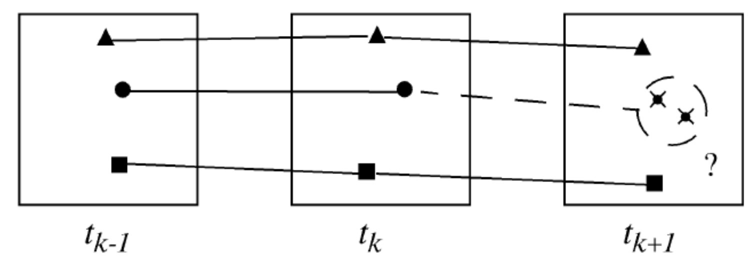
$$\left(x_k^1, y_k^1\right), \left(x_k^2, y_k^2\right), \cdots \left(x_k^N, y_k^N\right)$$

Definition of Particle Tracking (II)

- Different cases
  - Constant number of features
  - Feature appearance
  - Feature disappearance

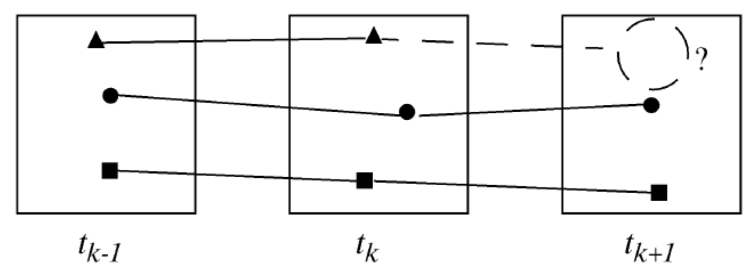


(a)



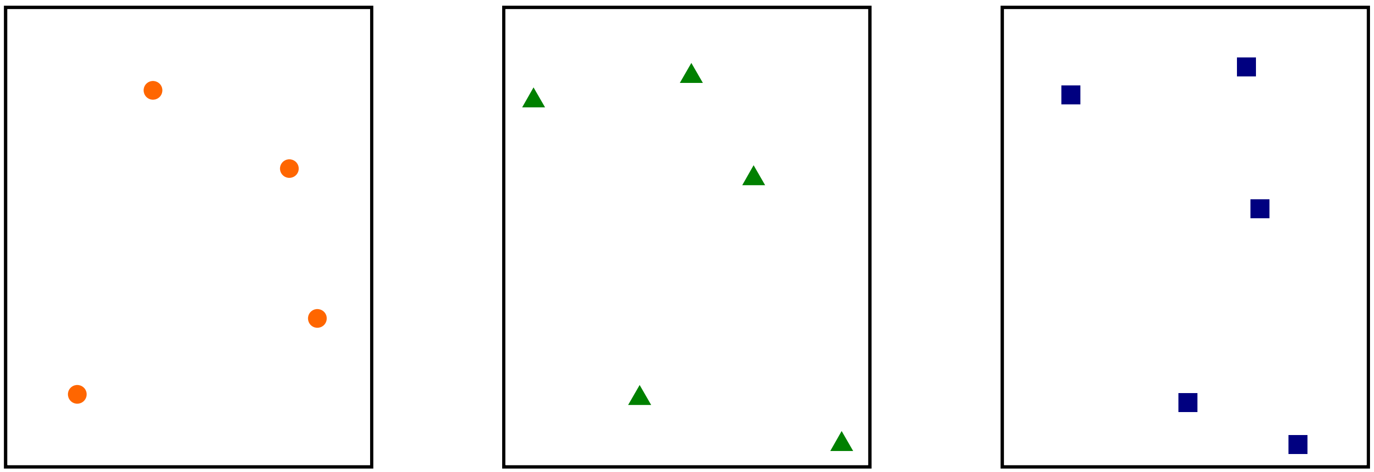
(b)

- Cases of feature appearance & disappearance
  - Moving in or out of field of view
  - Moving in or out of the focal plance
  - Assembly/disassembly
  - Feature merging/splitting



(c)

Example: Particle Tracking (I)



- Frame i-1
- Frame i
- Frame i+1

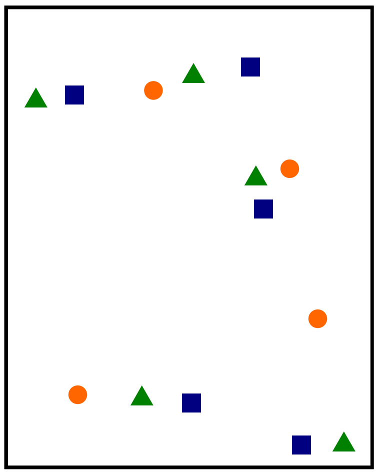
Example: Particle Tracking (II)

Frame i-1

Frame i

Frame i+1

Example of Particle Tracking (III)



- Frame i-1
- Frame i
- Frame i+1

Discussion: Different Tracking Strategies

- Strategy I: If the point correspondence between each pair of frames can be determined, the point correspondence over the entire image sequence is defined.
  - Advantages: relatively simple to implement
  - Disadvantages: a greedy approach, inadequate information to make a decision.

- Strategy II: to establish point correspondence based on information from multiple frames.
  - Advantages: decision making is more reliable.
  - Disadvantages: computationally intractable in most cases.

- Solution: to find a solution in between strategy I and II

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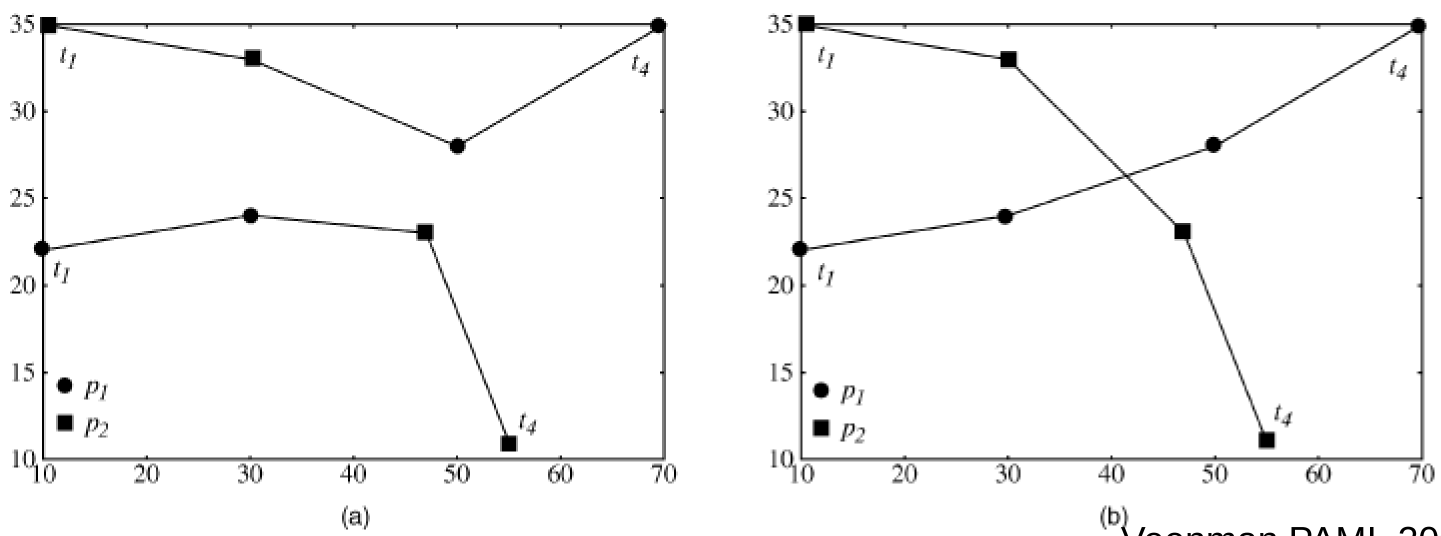
Particle Tracking Based on Global Linear Assignment (I)

- An optimization strategy is required to resolve conflicts between competing assignments.

- Selection of assignment weight will critically influence outcomes.



Veenman PAMI, 2001.

Particle Tracking Based on Global Linear Assignment (II)

- Formulation of the tracking problem as a bipartite graph assignment

$$min \sum_{i \in G_k} \sum_{j \in G_{k+1}} a^k(i,j) w^k(i,j)$$

$$st. \quad \sum_i a(i,j) = 1 \quad \sum_j a(i,j) = 1 \qquad a(i,j) \in \{0,1\}$$

- There are efficient numerical algorithms to solve large scale assignment problems.

- Why not use a tripartite graph?
  - Optimal assignment of tripartite graph is NP-complete.
  - Difficult to resolve conflicts between two tripartite assignments.

Commonly Used Assignment Weight Definitions

- Distance $\rightarrow$ Nearest neighbor

$$c^k(i,j) = \left\| x_j^{k+1} - x_i^k \right\|$$

- Smooth motion$\rightarrow$ Smooth motion

$$c^k(i,j) = w_1 \left[1 - \frac{\left(x_i^k - x_l^{k-1}\right)\left(x_j^{k+1} - x_i^k\right)}{\left\|x_i^k - x_l^{k-1}\right\| \left\|x_j^{k+1} - x_i^k\right\|} \right] + w_2 \left[1 - 2\frac{\sqrt{\left\|x_i^k - x_l^{k-1}\right\| \left\|x_j^{k+1} - x_i^k\right\|}}{\left\|x_i^k - x_l^{k-1}\right\| + \left\|x_j^{k+1} - x_i^k\right\|} \right]$$

- Mahalanobis distance, where the prediction comes from typically a Kalman filter

$$c^k(i,j) = \left(x_i^k - \hat{x}_i^k\right)^T S\left(x_i^k\right)^{-1} \left(x_i^k - \hat{x}_i^k\right)$$

How to Handle Particle Appearance & Disappearance

- Track appearance and disappearance are handled by introducing virtual points.

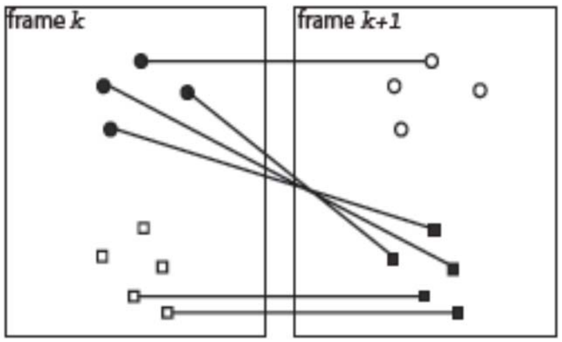


Figure 3. Handling particle appearance and disappearance

G. Yang, A. Matov, G. Danuser, Reliable tracking of large scale dense antiparallel particle motion for fluorescence live cell imaging, *IEEE CVPR, 2005*

References on Linear Assignment

- Schrijver A., *Combinatorial optimization*, vol. A, Chapter 17: Weighted bipartite matching and the assignment problem, pp.285-292, Springer, 2003.

- Burkard R., Amico M. D., Martello S., *Assignment problems*, SIAM, 2009.

- Burkard R., Cela E., Linear assignment problems and extensions, pp.75-149, in *Handbook of Combinatorial Optimization*, D.-Z. Du & P. M. Pardalos (Eds.), Kluwer Academic Publishers, 1999.

  (Downloadable from http://ccdl.compbio.cmu.edu/BME42_731/Burkard_LAP_review.pdf).

Questions?