Bioimage Informatics

Lecture 11, Spring 2012

Bioimage Data Analysis (III):

Edge Detection; Line/Curve Detection
Outline

- Review: low-level feature detection
- Overview of edge detection
- Line/curve detection using the Hough transform
• Review: low-level feature detection
  • Overview of edge detection
  • Line/curve detection using the Hough transform
Feature Detection: Points/Particles

Fluorescent speckles in a Xenopus extract spindle

Vesicles transported in a Drosophila motor neuron
Feature Detection: Lines/Curves

Video 1
(Figure 1A)

Microtubules in a PtK1 cell at the edge of an epithelial cell island.
Few microtubules rapidly grow into nascent protrusions.
Elapsed time: 9 min 05 sec


http://www.cell.com/cell_picture_show

Nikon Small World, 2003
Torsten Wittmann, UCSF
Filamentous actin and microtubules (structural proteins) in mouse fibroblasts (cells) (1000x)
Bovine pulmonary artery endothelial (BPAE) cells stained with a combination of fluorescent dyes. Mitochondria were labeled with red-fluorescent MitoTracker Red CMXRos, F-actin was stained using green-fluorescent Alexa Fluor 488 phalloidin, and blue-fluorescent DAPI was used to label the nuclei.
Feature Detection: Regions

Mitochondria in mouse hippocampal neuron, James Lim, LBNL

A neutrophil chasing a bacterium. Devreotes Lab, Johns Hopkins U.
Marr’s Theory of Visual Information Processing

• David Marr (Jan. 19, 1945 – Nov. 17, 1980)

• Three levels of cognition
  - Computational level
  - Algorithmic/representation level
  - Implementational level

• Three stages of vision
  - Primal sketch
  - 2.5D sketch
  - 3D model

Vision: A computational investigation into the human representation and processing of visual information
• Review: low-level feature detection

• **Overview of edge detection**

• Line/curve detection using the Hough transform
Edge Detection

• What is an edge?

An edge point, or an edge, is a pixel at or around which the image intensities undergo a sharp change.
Motivation: Edge in Images

- The edge can be treated as a 1D signal when examined in the normal direction.
Edge Can be Identified by Calculating Gradient

Gonzalez & Woods, DIP 3/e
How to Calculate Image Gradient

• Calculation of image gradient follows standard numerical differentiation scheme:

\[
\begin{align*}
  f(x+h) &= f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + O(h^3) \\
  f(x-h) &= f(x) - hf'(x) + \frac{1}{2}h^2f''(x) + O(h^3) \\
  f'(x_0) &= \frac{f(x_0+h) - f(x_0-h)}{2h} + O(h^2)
\end{align*}
\]

• In an image, the first and second derivatives:

\[
\begin{align*}
  \frac{\partial I(i,j)}{\partial x} &= I_x(i,j) = \frac{I(i+1,j) - I(i-1,j)}{2h} + O(h^2) \\
  \frac{\partial^2 I(i,j)}{\partial x^2} &= I_{xx}(i,j) = \frac{I(i+1,j) - 2I(i,j) + I(i-1,j)}{h^2} + O(h)
\end{align*}
\]
Gradient Calculation is Sensitive to Noise

- Without image smoothing, calculation of derivatives becomes highly sensitive to noise.

Gonzalez & Woods, DIP 3/e
Edge Detection Procedure

- Step I: noise suppression
- Step II: edge enhancement
- Step III: edge localization

Why Use Gaussian Kernel for Smoothing

- Gaussian kernel is not the only smoothing kernel.

- It has several important advantages:
  - Convolution of a Gaussian with another Gaussian is Gaussian.
  - Efficiency. Gaussian kernel is separable.
  - Repeated smoothing with a low-pass filter will eventually converge to Gaussian smoothing.
Combination of Noise Suppression and Gradient Estimation (I)

• Notation:
  - \( J \): raw image;
  - \( I \): filtered image after convolution with Gaussian kernel \( G \).

• A basic property of convolution

\[
\begin{align*}
\frac{\partial (G \ast J)}{\partial x} &= \frac{\partial I}{\partial x} = I_x = \frac{\partial G}{\partial x} \ast J \\
\frac{\partial (G \ast J)}{\partial y} &= \frac{\partial I}{\partial y} = I_y = \frac{\partial G}{\partial y} \ast J
\end{align*}
\]

\( E_s(x_0, y_0) = \sqrt{I_x^2(x_0, y_0) + I_y^2(x_0, y_0)} \) ← Edge strength

\( E_o(x_0, y_0) = \arctan \frac{I_y(x_0, y_0)}{I_x(x_0, y_0)} \) ← Edge orientation
Combination of Noise Suppression and Gradient Estimation (II)

- Gaussian kernel in 1D

\[ G(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}} \]

- First order derivative

\[ G'(x) = \frac{-x}{\sqrt{2\pi\sigma^3}} e^{-\frac{x^2}{2\sigma^2}} \]

- Second order derivative

\[ G''(x) = \frac{-x}{\sqrt{2\pi\sigma^3}} e^{-\frac{x^2}{2\sigma^2}} \left[ 1 - \frac{x^2}{\sigma^2} \right] \]
Combination of Noise Suppression and Gradient Estimation (III)

• Implementation

\[
G(x, y; \sigma_x, \sigma_y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp \left\{ -\frac{1}{2} \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right) \right\} = \frac{1}{\sqrt{2\pi \sigma_x}} \exp \left\{ -\frac{1}{2} \frac{x^2}{\sigma_x^2} \right\} \cdot \frac{1}{\sqrt{2\pi \sigma_y}} \exp \left\{ -\frac{1}{2} \frac{y^2}{\sigma_y^2} \right\} = G(x, \sigma_x) \cdot G(y, \sigma_y)
\]

\[
\frac{\partial G(x, y; \sigma_x, \sigma_y, \theta)}{\partial x} = \frac{d G(x, \sigma_x)}{d x} G(y, \sigma_y) \quad \frac{\partial G(x, y; \sigma_x, \sigma_y, \theta)}{\partial y} = G(x, \sigma_x) \frac{d G(y, \sigma_y)}{d y}
\]

• Advantages:

- Reduced computational cost
- Calculation of gradient can run in parallel in both directions
Edge Enhancement

• Step I: For each pixel $I(x_0, y_0)$, calculate the gradient

$$\left. \frac{\partial I}{\partial x} \right|_{x=x_0} \quad \left. \frac{\partial I}{\partial y} \right|_{y=y_0}$$

• Step II: Estimate edge strength

$$E_s(x_0, y_0) = \sqrt{I_x^2(x_0, y_0) + I_y^2(x_0, y_0)}$$

• Step III: Estimate edge direction

$$E_o(x_0, y_0) = \arctan \frac{I_y(x_0, y_0)}{I_x(x_0, y_0)}$$
Calculation of Image Gradient
Non-Maximum Suppression

• For each pixel $I(x_0,y_0)$, compare the edge strength along the direction perpendicular to the edge

• An edge point must have its edge strength no less than its two neighbors.
Hysteresis Thresholding

• The main purpose is to link detected edge points while minimizing breakage.

• Basic idea
  - Using two thresholds $T_L$ and $T_H$
  - Starting from a point where edge gradient magnitude higher than $T_H$
  - Link to neighboring edge points with edge gradient magnitude higher than $T_L$
Influence of Scale Selection on Edge Detection

Edge Detection Demo
• Review: low-level feature detection

• Overview of edge detection

• Line/curve detection using the Hough transform
Line/Curve Detection by

• A variety of techniques are available.

  - Spatial domain
    - e.g. by edge point detection and grouping

  - Transform domain
    - e.g. by Hough transform
Basic Concept of Hough Transform

• A simple example: representing the lines passing through \((x_0, y_0)\) in the parameter space.

\[ y_1 = a \cdot x_1 + b \quad y_2 = a \cdot x_2 + b \]

\[ b = (-x_1) a + y_1 \quad b = (-x_2) a + y_2 \]

\[ y_1 = \tilde{a} \cdot x_1 + \tilde{b} \quad y_2 = \tilde{a} \cdot x_2 + \tilde{b} \]

The two lines in the transform domain must intersect at \((a, b)\)
HT Algorithm Implementation Details

- Parameterization using $y=mx+n$ fails for the case of vertical lines.

- A different way of parameterization:
  \[ \rho = x \cdot \cos \theta + y \cdot \sin \theta \]

- Exhaustive search the space of $[\rho, \theta]$ can be time-consuming.
Generalization of the HT Algorithm for Curve Detection

• The HT algorithm is a voting algorithm. The key idea is to convert a (difficult) pattern recognition problem into a (simple) peak detection problem.

• Hough transform can be generalized to detect circles, ellipses, or any curve that can be parameterized.

• Examples

  Circles with known radius but unknown center

  \[(x - x_c)^2 + (y - y_c)^2 = R^2 \quad \Rightarrow \quad (x_c - x_i)^2 + (y_c - y_i)^2 = R^2\]

  Ellipses with known major and minor semi-axes but unknown center

  \[\frac{(x - x_c)^2}{a^2} + \frac{(y - y_c)^2}{b^2} = 1 \quad \Rightarrow \quad \frac{(x_c - x_i)^2}{a^2} + \frac{(y_c - y_i)^2}{b^2} = 1\]
Evaluation of Parametric Transform Based Curve Detection

• The curve to be detected can be of arbitrary form as long as it can be parameterized.

• **Strengths:**
  - Handles occlusion and partial line/curves well.
  - Relatively robust to noise
  - Capable of detecting multiple instances

• **Limitations**
  - For curves with multiple parameters, the voting/search can be costly.
  - Other shapes can also generate spurious peaks.
Comments on Line/Curve Detection

- Curves features are very common in bioimages.

- General curve feature detection will be addressed in the next lecture.
Questions?