33-765 — Statistical Physics

Department of Physics, Carnegie Mellon University, Spring Term 2019, Deserno

Problem sheet #8

26. Examples of simple thermodynamic identities *(4 points)*

Rewrite the following thermodynamic derivatives until only the "standard" derivatives α_P, κ_T, c_P, and c_V (and possibly factors, such as T or P, or numbers, such as 2 or π) occur:

1. $\left(\dfrac{\partial T}{\partial P}\right)_{S,N} \;=\; ?$

2. $\left(\dfrac{\partial F}{\partial S}\right)_{T,N} \;=\; ?$

27. Maxwell relations and Jacobians in tedious disguise *(4 points)*

1. Prove the first $T\,\mathrm{d}S$ equation: $\quad T\,\mathrm{d}S = Nc_V\,\mathrm{d}T + \dfrac{\alpha T}{\kappa_T}\,\mathrm{d}V$.
2. Prove the second $T\,\mathrm{d}S$ equation: $\quad T\,\mathrm{d}S = Nc_P\,\mathrm{d}T - \alpha TV\,\mathrm{d}P$.

Hint: If you write the entropy $S(\cdot,\cdot,N)$ in the variables indicated by the $T\,dS$ equation(s), what would be its differential?

28. Changing the chemical potential *(4 points)*

If one changes the chemical potential of a particle reservoir, the number of particles in one's system of interest will change as well. We can measure this by some partial derivatives which, for lack of a better name, I will hereby call "fillability":

$$\varphi_{S,V} \;:=\; \frac{1}{N}\left(\frac{\partial N}{\partial \mu}\right)_{S,V} \qquad \text{or} \qquad \varphi_{T,V} \;:=\; \frac{1}{N}\left(\frac{\partial N}{\partial \mu}\right)_{T,V} .$$

The first one could be called the "adiabatic fillability," and the second one the "isothermal fillability."

If $C_{N,V}$ and $C_{\mu,V}$ are heat capacities (with the indicated variables fixed, and *without* making them specific by dividing by N), prove the following thermodynamic identity:

$$\frac{\varphi_{S,V}}{\varphi_{T,V}} \;=\; \frac{C_{N,V}}{C_{\mu,V}} .$$

29. Relation between the isothermal and the adiabatic compressibilities *(4 points)*

In analogy to the well-known relation between the isobaric and the isochoric heat capacities, c_P and c_V, derive the following very similar formula for the isothermal and adiabatic compressibilities, κ_T and κ_S:

$$\kappa_T - \kappa_S \;=\; \frac{TV\alpha^2}{Nc_P} . \tag{1}$$

30. Adiabatic compression *(4 points)*

1. Show that, quite generally, $\dfrac{\kappa_T}{\kappa_S} = \dfrac{c_P}{c_V} =: \gamma$, where γ is called the *adiabatic index*.

2. Calculate κ_T, c_V, c_P, and γ for the monoatomic ideal gas.

3. Show that for adiabatic (constant entropy) compression of an ideal gas we get $P \propto V^{-\gamma}$.