33-341 — Thermal Physics I

Department of Physics, Carnegie Mellon University, Fall Term 2018, Deserno

Sample Midterm 2 Questions

1. Isothermal and adiabatic compressibilities

In analogy to the relation between c_P and c_V, derive an analogous formula for the two compressibilities:

$$\kappa_T - \kappa_S = \frac{TV\alpha^2}{Nc_P} . \tag{1}$$

2. Re-expressing thermodynamic derivatives

Express the following thermodynamic derivative in terms of the standard set of derivatives (and possibly other factors, such as temperature, or pressure, or π):

$$\left(\frac{\partial F}{\partial S}\right)_{T,N} = \ ?$$

3. Legendre transform

Consider the function $f(x) = \sqrt{1-x^2}$ with $x \in [-1, 1]$.

1. Produce a *clean* sketch of $f(x)$. Pay particular attention to zeros, extrema, symmetry, and asymptotes.
2. What range of values can the slope $f'(x)$ take?
3. Is $f(x)$ convex, concave, or neither?
4. Calculate the Legendre transform $f^\star(p)$ of $f(x)$. What is the domain of p-values on which $f^\star(p)$ is defined?
5. Now make a *clean* sketch of $f^\star(p)$. Again, pay particular attention to zeros, extrema, symmetry, and asymptotes.

4. Inexact differentials and integrating factors

1. Show that for all $\beta \in \mathbb{R}$, *with one exception*, the differential $đf = \sqrt{\frac{y}{x}}\,\mathrm{d}x - \beta\sqrt{\frac{x}{y}}\,\mathrm{d}y$ is not exact. What is the exception?
2. Show that for all $\beta \in \mathbb{R}$, *with one exception*, there is an $\alpha \in \mathbb{R}$ for which $r(x,y) = \left(\frac{x}{y}\right)^\alpha$ becomes an integrating factor. What is the exception?

5. Work done between two finite heat reservoirs

This is similar to HW problem 29, but with a slight twist. Suppose we have two finite heat reservoirs with *temperature-dependent* heat capacities $C_i(T) = A_i T^3$ ($i \in \{1, 2\}$), so that the relationship between their change in temperature and their change in energy is given by $\mathrm{d}U_i = C_i(T)\,\mathrm{d}T$. The reservoirs are initially at temperature $T_{i,0}$. We now put an ideal heat engine between them, depleting that temperature difference to extract mechanical work.

1. What is the final temperature T_f?
2. What is the maximum amount of work that can be extracted?
3. If we bring the reservoirs into contact without running the machine, what is now the final temperature $T_\mathrm{f}^\star$?