33-341 — Thermal Physics I

Department of Physics, Carnegie Mellon University, Fall Term 2018, Deserno

Problem sheet #14

41. Ice skating *(5 points, due on Monday)*

First a few facts:

1. Icebergs consist of freshwater, and only about 10% of an iceberg is visible above the ocean's surface.
2. The density of sea water is about $1.025\,\mathrm{g/cm^3}$.
3. It takes about $334\,\mathrm{kJ}$ to melt one kilogram of ice (from just below freezing to just above freezing).

Now here comes the problem you will be able to solve using these data:

1. What is the slope of the melting curve in the T-P diagram of water at atmospheric pressure?
2. Estimate the area under an ice skate.
3. Most ice rinks operate at about $-7\,^\circ\mathrm{C}$. How heavy would a person have to be so that the pressure exerted on the ice through the blades of that person's skates will pressure-melt the ice?

42. Approximate shape of the liquid-gas coexistence curve *(5 points, due on Tuesday)*

Far away from the critical point of any substance, the specific volume (*i. e.*, the volume per particle) of its gas phase is *much* larger than the specific volume of its liquid phase, and so the latter can be ignored in the Clausius-Clapeyron equation. For the gas, let us further assume the ideal gas law to hold.

1. Derive a differential equation for the liquid-gas coexistence pressure as a function of temperature. This will involves the generally temperature-dependent *specific latent heat* $L(T)$ (*i. e.*, the latent heat per particle).
2. If $L(T) \approx L =$ const., integrate the differential equation.

43. Boiling eggs in Denver *(5 points, due on Wednesday)*

Water boils when the pressure on the liquid-gas coexistence curve exceeds atmospheric pressure. At *standard* atmospheric pressure, this happens at about 212°F, but as we go to higher altitudes, this changes, because atmospheric pressure changes.

1. Recalling problem 40, predict the average atmospheric pressure in Denver, famously called the "mile high city".
2. The latent heat of evaporation of water is about $2230\,\mathrm{J/g}$. Using your result from problem 42.2, predict the temperature at which water boils in Denver!
   Hint: Water has a molecular mass of $18\,\mathrm{g/mol}$, and $1\,\mathrm{mol} \approx 6.022 \times 10^{23}$.
3. Produce a nice plot of the boiling point of water as a function of height, all the way up to $10\,\mathrm{km}$. Mark Denver, La Paz (the highest capital city in the world), and the top of Mt. Everest in your plot.

44. The equipartition theorem *(5 points, due on Friday)*

Consider a Hamiltonian $H(\{p,q\})$ on phase space. Let x_i be any of the $6N$ coordinates, for instance, it could be $p_{27,y}$ or $q_{1673536,x}$. Let $\langle\,\cdot\,\rangle$ denote the canonical average (*i. e.*, the average over the canonical state).

1. Prove that the following is true: $\left\langle x_i \frac{\partial H}{\partial x_j}\right\rangle = k_\mathrm{B}T\,\delta_{ij}$.
   Hint: first calculate $\frac{\partial}{\partial x_j}\mathrm{e}^{-\beta H}$. Also: integration by parts (you may assume the boundary term vanishes).
2. If the Hamiltonian contains a term Ax^n, and this is the only occurrence of x, prove that $\langle Ax^n\rangle = \frac{1}{n}\,k_\mathrm{B}T$.
3. For the "standard" kinetic energy, and p_i one of the (scalar) momentum coordinates, prove that $\left\langle \frac{p_i^2}{2m}\right\rangle = \frac{1}{2}\,k_\mathrm{B}T$.