33-341 — Thermal Physics I

Department of Physics, Carnegie Mellon University, Fall Term 2018, Deserno

Problem sheet #11

34. Deflategate *(5 points, due on Monday)*

The football game between the New England Patriots and the Indianapolis Colts, which took place in Foxborough, MA, on the evening of January 18th 2015, achieved quite some national notoriety that suddenly had everyone talk about physics and pressure gauges. The story is as follows: the footballs used by the Patriots during the first half were found to be significantly under-inflated: measurements gave a mean of 11.3 psi, which is 1.2 psi below the minimum permissible value (which ranges between 12.5 psi and 13.5 psi). It was furthermore widely known that their quarterback, Tom Brady, preferred to play with low-pressurized footballs. However, instead of admitting to any type of wrongdoing, the Pats blamed thermodynamics for this. More specifically, they claimed that they had properly inflated their balls in their locker room before the game, but due to a change in weather conditions towards halftime, the pressure wasn't what it was supposed to be. Let's figure out whether this could be true. By how much would you expect the gauge pressure to have dropped by half-time due to local weather conditions?

Note: Historical weather data (which at the time of the incident were reported and amply discussed for weeks…) show that the halftime temperature (around 8:30pm) in Foxborough, MA, was about 52 F, and the barometric pressure had dropped from 1010 hPa before the game to about 1004 hPa at halftime.

If you want to practice for the midterm, have a look at the sample questions on the course webpage!

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35. Adiabatic oscillations *(5 points, due on Friday)*

Imagine a cylinder of cross-sectional area A. It stands on its permanently closed bottom, is filled with some gas (not necessarily an ideal one), and closed at the top by a tightly fitting lid of mass M that can *slide vertically without friction* along the inside walls of the cylinder. In equilibrium, the lid will rest at some height h above the bottom, enclosing a volume $V = Ah$.

1. List all the forces that act on the lid. Which directions do they have? Which of them depend on the lid's vertical position?
2. If you displace the lid away from equilibrium and then release it, it will oscillate with some frequency ω. Assume that the container walls and the lid conduct heat very poorly, hence the periodic compressions and expansions of the gas happen adiabatically. Derive a differential equation for the motion of the lid, valid for small oscillations. What is the frequency?
3. If the cylinder contains a monoatomic ideal gas, show that the frequency can be written as $\omega^2 = \frac{5/3}{Mh}(P_\mathrm{o}A + Mg)$, where P_o is the outside air pressure and g the acceleration of gravity.

   Notice that you could use this device to measure the air pressure P_o, namely, by measuring the oscillation frequency!