33-341 — Thermal Physics I

Department of Physics, Carnegie Mellon University, Fall Term 2018, Deserno

Problem sheet #2

4. Rolling dice *(7 points, due on Tuesday)*

1. Let us consider rolling an ordinary 6-sided die. As outcome, we care about the number E of eyes showing up. What is the expectation value $\langle E \rangle$ and the variance σ_E^2 of the random variable E?

2. Generalize your results to a die that has S sides, each of which shows up with the same probability.
The formulas $\sum_{k=1}^N k = \frac{1}{2}N(N+1)$ and $\sum_{k=1}^N k^2 = \frac{1}{6}N(N+1)(2N+1)$ will come in handy.

3. If we roll N of these S-dice, then we can define the random variable E_N as the total number of eyes showing up. Calculate $\langle E_N \rangle$ and the variance $\sigma_{E_N}^2$.

4. Download the program `roll-multiple-dice.py` from the course webpage. It simulates repeatedly rolling some number of S-dice `number_of_trials` times and records a histogram of the outcome, which we can use as an estimate for the probability distribution of E_N. However, the program will not run error-free until you complete the two lines which start with `mean=...` and `var=...` such that they calculate the expectation value and the variance of the random variable E_N, for given values of $N =$ `number_of_dice` and $S =$ `sides_on_die`. Do that and then run the program. The program in particular compares the estimate of the probability distribution of E_N with a Gaussian distribution function of the same mean and variance (see textbook, Sec. 3.9). What do you observe as you change N, S, and the number of trials? Append some characteristic plots to your homework.

5. Birthdays *(7 points, due on Wednesday)*

1. What is the probability P_N that in a group of N people at least 2 have their birthday on the same day? For simplicity, let us assume two things: first, a year has 365 days; that is, let us ignore leap year complications. And second, people are born with equal probability on any day of the year.
This is a good example where calculating the probability of the opposite event is much easier!

2. Produce a table that lists N and P_N for values of $N \in \{1, 2, 3, \ldots, 50\}$.
This is an excellent occasion for a little Python program: calculating P_N for all these different values of N is tedious, so let's have a computer do this task for you! If you do that, also hand in the code you wrote.

3. What's the smallest number of people for which P_N is bigger than 50%?

4. The exact formula you worked out in part (1) can be written (even though not easily *calculated*!) using factorials. Using Gosper's version of Stirling's approximation—formula (3.73) in your textbook—to approximate these factorials, you can get a nicer "closed" expression for P_N. List its prediction next to the exact results. How good is it?
Be prepared to calculate about 12 digits behind the decimal point!

6. A famous probability "paradox" *(6 points, due on Friday)*

Here's a very famous problem pertaining to the topic of independent events and conditional probabilities, which exemplifies how crucially important it can be precisely how a given piece of statistical information is being conveyed.

1. Mrs. Jones is the mother of two children. You happen to know that at least one of them is a boy. What are the odds that Mrs. Jones has a daughter?

2. Mrs. Muller is the mother of two children, but you don't know how many boys and girls she has. One day you encounter her on the street accompanied by a boy, who she proudly introduces to you as her son. What are the odds that Mrs. Muller has a daughter?
Hint: Bayes' formula comes in handy!

3. Explain why this set of questions is perceived (by many) as paradoxical. What is the "solution"?

Comment: This problem can be confusing enough without additional nitty-gritty complications. So let us assume that (to a very good approximation) the two sexes are equiprobable, and that the sexes of any two children (born separately) are independent. So let's not waste time squabbling about human sex ratios slightly different from 1:1, or any twin complications!