## Problem sheet \#1

## 1. Getting started (5 points, due on Tuesday)

1. Log onto the course webpage (http://www. andrew. cmu. edu/course/33-341/) and read the syllabus.
2. Get hold of a version of Python that you can run. I recommend downloading Anaconda (free!), which is a distribution that contains the Python language (version 3.6), the interpreter iPython, the comfortable integrated development environment (IDE) Spyder, and various useful libraries, such as NumPy, PyPlot, and SciPy.
3. Run the program OneDie_PARTIAL. py, which you find on the course webpage. It simulates the rolling of a die for trials times, and prints out the number of occurrences of each face, followed by the absolute and relative deviation from the expected value. Change the parameter trials; describe and document (with data!) what you observe. Note: don't worry if you do not understand all of the code immediately. Python is a very intuitive language which you can learn as you go along. Often it suffices to change existing programs and adapt them to your needs. Also, you can very easily google how other things are done.

## 2. Playing games with non-standard dice (8 points, due on Wednesday)

The picture on the right shows a curious quadruplet of dice: their faces are not labeled consecutively with the numbers one to six, but rather have the following numbers printed on them:

| $D_{1}:$ | $\{3,3,3,3,3,3\}$ | (yellow) |
| :--- | :--- | :--- |
| $D_{2}:$ | $\{0,0,4,4,4,4\}$ | (red) |
| $D_{3}:$ | $\{1,1,1,5,5,5\}$ | (green) |
| $D_{4}:$ | $\{2,2,2,2,6,6\}$ | (blue) |

We will assume that, like for an ordinary die, the likelihood of any of the six sides of any of the four dice coming up when we roll the die is $1 / 6$.
Assume now that two people are playing a game: each one picks one of the dice and then they roll them. Whoever gets the higher number wins the game. Since the dice are different, it is conceivable that the probability of winning depends on which die one chooses. There is no reason to believe it is still $50 \%$, as it would be for normal dice!


Hence, let us introduce the following terminology: We call die $A$ "stronger" than die $B$ (or die $B$ "weaker" than die $A$ ), when the probability of a roll of $A$ beating a roll of $B$ is larger than $50 \%$. Let us formally write this as " $A>B$ ", or " $B<A$ ".

1. Show that $D_{1}<D_{2}, D_{2}<D_{3}$, and $D_{3}<D_{4}$ !

Hint: Of course you can do this by carefully calculating the probabilities of winning, but you don't have to do it that way. In all cases it is possible to give a simple (but stringent!) argument to show why one die is stronger than the other.
2. Based on the sequence you have just shown, do you expect $D_{4}>D_{1}$ ? Check explicitly whether this holds!
3. How does $D_{1}$ compare to $D_{3}$, and how does $D_{2}$ compare to $D_{4}$ ?

## 3. Pairwise and mutual independence (7 points, due on Friday)

We flip a fair coin twice and see whether it falls head ("H") or tail ("T"). Let us look at the following three events: A: "There was a head in the first throw." B: "There was a head in the second throw." C: "The result was the same in the first and the second throw." Show that the three events $\mathrm{A}, \mathrm{B}$ and C are pairwise independent but not mutually independent!

