A counter is placed behind a circular collimator of diameter 1.00 cm located 20.0 cm from a scattering target which is an aluminum foil mounted at 45° to the incident beam direction, see figure. The foil has a thickness such that 1 cm² weights 5.00 × 10⁻³ g.

Suppose that the beam consists of 12 MeV protons with a current of 1.00 × 10⁻⁷ amps, and that the counter registers 250 counts/s. Calculate the differential cross section $d\sigma/d\Omega$ (in the laboratory) for scattering from the aluminum nucleus, in cm²/steradian, at an angle of 90° (as shown in the figure).
The foil has a mass of 5.00 mg/cm\(^2\) and thus contains

\[
5.00 \times 10^{-3} \times 6.022 \times 10^{23} / 26.98 = 1.116 \times 10^{20} \text{ atoms/cm}^2.
\]  
(1)

However, as it is inclined at 45° to the incident beam direction, its thickness is effectively multiplied by \(1/\cos(45°) = \sqrt{2}\), thus giving

\[
\rho' = 1.578 \times 10^{20} \text{ atoms/cm}^2.
\]  
(2)

The collimator subtends a solid angle of

\[
\Delta\Omega = \pi \left(\frac{1}{2}\right)^2 / 20^2 = \pi / 1600 = 1.963 \times 10^{-3} \text{ str},
\]  
(3)

If \(\sigma'(\Omega) = d\sigma/d\Omega\) is the differential cross section, the probability that a particle in the incident beam will be scattered into the counter is

\[
P = \rho'\sigma'\Delta\Omega = I_s / I_0,
\]  
(4)

where the beam current of \(10^{-7}\)amps corresponds to

\[
I_0 = 10^{-7} / 1.602 \times 10^{-19} = 6.24 \times 10^{11} \text{ particles s}^{-1},
\]  
(5)

and the current in the detector is \(I_s = 250 \text{ particles s}^{-1}\). Hence the differential cross section at 90° is given by

\[
\sigma' = d\sigma / d\Omega = \frac{250}{(6.24 \times 10^{11}) \cdot (1.578 \times 10^{20}) \cdot (1.963 \times 10^{-3})}
\]  
\[= 1.293 \times 10^{-27} \text{ cm}^2 = 1.293 \times 10^{-3} \text{ barns}.
\]  
(6)