33-331 Physical Mechanics I. Fall Semester, 2008
Hour Examination No. 3, 17 November 2008.

NAME______________________________

INSTRUCTIONS. This examination consists of two problems. The first, with 3 parts, is worth 60 points, and the second, with 2 parts, is worth 40 points. *It is important to give a brief indication of your reasoning.* Answer in a way which shows that you *understand* the subject. If you do not understand what a question means, or think additional formulas would be helpful, please ask the instructor.

TR-0330 Hamiltonian, phase space for vertical motion in gravity

F08 exm3.1 60pts

1. (3 parts, total of 60 points) A particle of mass m moves vertically in a uniform gravitational field. Its potential energy is $U = mgy$, y is the height measured vertically upwards, and p is the vertical momentum, positive when the particle is moving upwards.

a) Write down the Hamiltonian—you do not need to derive it, though you may if you wish—as a function of y and p (treat this as a one-dimensional problem). Then employ Hamilton's equations to derive a differential equation for the acceleration $\ddot{y}$. Show your work.

b) Find a formula for y as a function of p along a trajectory in the y, p phase plane. [Hint: What is dy/dp along such a trajectory? Or you can find the formula if you know both $y(t)$ and $p(t)$.]

c) Sketch two such trajectories in the y, p plane, with y horizontal and p vertical, using both positive and negative values of p. Indicate by an arrow or arrows on each trajectory the direction the phase point moves as time increases. Now consider the region R in the phase plane bounded by your two trajectories and two horizontal lines, $p = p_1$ and $p = p_2$, with $0 < p_1 < p_2$. What happens to this region as time increases from $t = 0$ to some later time $t = \tau$? (I. e., as each point in the initial region, thought of as representing the possible state of a system at $t = 0$, moves according to the equations of motion to a later time $t = \tau$.) Relate this to Liouville's theorem. [Hint: Something very simple happens as time progresses to a line in the phase plane whose initial position is defined by p =constant.]

continued on next page

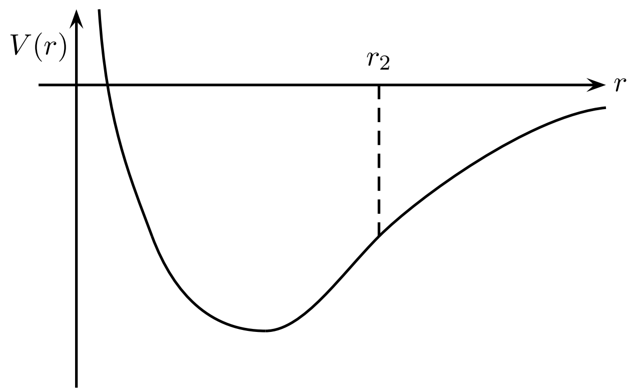
TR-0415 Central potential: ratios of quantities at $r_{\min}$ and $r_{\max}$

F08 exm3.2 40 pts

2. (2 parts, total of 40 points) For a particle moving in an attractive central potential $U(r)$ the effective potential

$$V(r) = U(r) + l^2/2\mu r^2$$

for a particular angular momentum $l = \mu r^2\dot{\theta}$ is as shown in the sketch. The particle moves in a bound orbit, not necessarily an ellipse, with r_1 the minimum and r_2 the maximum distance of approach to the center. In polar coordinates the kinetic energy is $T = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2)$.



a) Indicate how by using the sketch of $V(r)$ you can obtain r_1 given the value of r_2 and $V(r)$. Explain what you are doing. Are any conservation laws involved?

b) Let $\lambda < 1$ be the ratio r_1/r_2, let $\dot{\theta}_1$ and $\dot{\theta}_2$ be the angular velocities when $r = r_1$ and r_2, and T_1 and T_2 the corresponding kinetic energies. Find the ratios $\dot{\theta}_1/\dot{\theta}_2$ and T_1/T_2 in terms of λ and possibly other quantities. For the T_1/T_2 ratio start from the fact that in polar coordinates $T = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2)$. Indicate your reasoning.