33-331 Physical Mechanics I. Fall Semester, 2009
Assignment No. 9
Due Friday, November 6

READING

Thornton and Marion Ch. 8, Secs. 8.7, 8.9, 8.10: Kepler problem, apsidal angles, nearly circular orbits

Handout on hyperbolic orbits

READING AHEAD:

Thornton and Marion Ch. 8, Sec. 8.8: Hohmann transfer, slingshot

Excerpt from Physics Today on slingshot

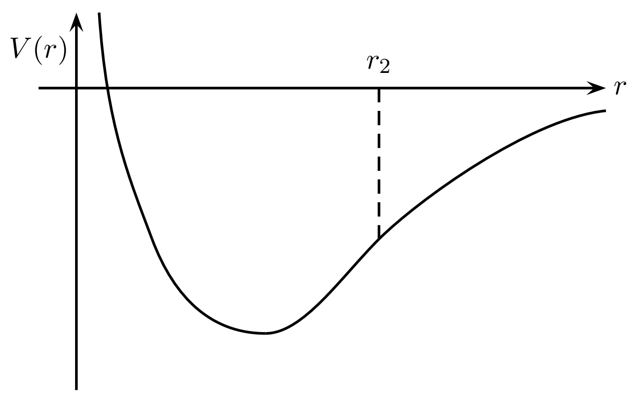
EXERCISES

1. Turn in at most one page, and not less than a third of a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc.

2. (Fall 2008, third hour exam) For a particle moving in an attractive central potential $U(r)$ the effective potential

$$V(r) = U(r) + l^2/2\mu r^2$$

for a particular angular momentum $l = \mu r^2\dot{\theta}$ is as shown in the sketch. The particle moves in a bound orbit, not necessarily an ellipse, with r_1 the minimum and r_2 the maximum distance of approach to the center. In polar coordinates the kinetic energy is $T = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2)$.



a) Indicate how by using the sketch of $V(r)$ you can obtain r_1 given the value of r_2 and $V(r)$. Explain what you are doing. Are any conservation laws involved?

b) Let $\lambda < 1$ be the ratio r_1/r_2, let $\dot{\theta}_1$ and $\dot{\theta}_2$ be the angular velocities when $r = r_1$ and r_2, and T_1 and T_2 the corresponding kinetic energies. Find the ratios $\dot{\theta}_1/\dot{\theta}_2$ and T_1/T_2 in terms of λ and possibly other quantities. For the T_1/T_2 ratio start from the fact that in polar coordinates $T = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2)$. Indicate your reasoning.

3. Thornton and Marion 8-3

4. Thornton and Marion 8-11

5. Thornton and Marion 8-23

6. Thornton and Marion 8-25

7. The alien spacecraft is approaching the solar system on a trajectory that will pass the sun at the radius of Mercury. At a very great distance its speed towards the sun is 10,000 ms^{-1}. What will its speed be at the point of closest approach to the sun?