33-231 **Physical Analysis** **Fall 2003**

Problem Solutions: Set 1 (Wednesday, September 3, 2003)

1. Introduction to Maple exercises

2. a) $m\dfrac{dv}{dt} = -bv^2.$

$$v = \frac{v_o}{1 + v_o \dfrac{b}{m} t}.$$

b) $$x = x_o + \frac{m}{b}\ln\frac{\dfrac{m}{bv_o} + t}{\dfrac{m}{bv_o}} = x_o + \frac{m}{b}\ln\left(1 + \frac{bv_o}{m}t\right).$$

c) When $\dfrac{v_o}{e} = \dfrac{v_o}{1 + v_o \dfrac{b}{m} t}$, $\quad t = (e-1)\dfrac{m}{v_o b}$. At this time,

$$x = x_o + \frac{m}{b}\ln\left[1 + \frac{bv_o}{m}(e-1)\frac{m}{v_o b}\right] = x_o + \frac{m}{b}.$$

d) $$b = \frac{F}{v^2} = \frac{20\ \mathrm{N}}{(1\ \mathrm{m/s})^2} = \frac{20\ \mathrm{kg\ m/s^2}}{(1\ \mathrm{m/s})^2} = 20\ \ \mathrm{kg/m}.$$

From (c), m/b must have units of distance (i.e., meters), and it does.

e) If $x_o = 0$, $\quad m = 100$ kg, $\quad b = 20$ kg/m, then from (c),

$$t = (e-1)\frac{m}{v_o b} = (e-1)\frac{100\ \mathrm{kg}}{(2.0\ \mathrm{m/s})(20\ \mathrm{kg/m})} = 4.3\ \mathrm{s},$$

$$x = x_o + \frac{m}{b} = x_o + \frac{100\ \mathrm{kg}}{20\ \mathrm{kg/m}} = x_o + 5.0\ \mathrm{m}.$$

3. a) $N = N_o e^{-\lambda t}$.

b) $\dfrac{N_o}{2} = N_o e^{-\lambda t_{1/2}}$, $\ln\frac{1}{2} = -\lambda t_{1/2}$, $t_{1/2} = \dfrac{\ln 2}{\lambda}$.

c) $\dfrac{N_o}{e} = N_o e^{-\lambda t}$, $\ln\dfrac{1}{e} = -\lambda t$, $\tau = \dfrac{1}{\lambda}$.

4. a) Number of disintegrations per unit time is dN/dt. From Problem 3(a),

$\dfrac{dN}{dt} = -\lambda N_o e^{-\lambda t} = -\lambda N$. The activity is the magnitude of this quantity. As N decreases, the activity decreases.

b) Atomic mass is 226 g/mol; number of moles is (1 g)/(226 g/mol) = 1/226 mol. Number N of atoms is Avogadro's number (number of atoms in a mole) multiplied by number of moles:

N = (6.02×10^{23} atoms/mol)(1/226 mol) = 2.66×10^{21} atoms.

$t_{1/2}$ = 1600 yr = 5.06×10^{10} s. From (b), $\lambda = \dfrac{\ln 2}{t_{1/2}} = 1.37 \times 10^{-11}\ \text{s}^{-1}$.

Activity = λN = (2.66×10^{21})(1.37×10^{21}) = 3.65×10^{10} decays/s.

1 Ci = 3.65×10^{10}Bq.

c) Each decay releases 4.87 MeV. Also, 1 MeV = 1.60×10^{-13} J, so each decay releases (4.87 MeV)(1.60×10^{-13} J/MeV) = 7.80×10^{-13} J.

From above, there are 3.65×10^{10} decays per second, so the total energy released per second by 1 g of Radium is

(7.80×10^{-13} J)(3.65×10^{10}/s) = 0.0285 J/s = 0.0285 W = 28 mW.

d) Calorimetric relation: $Q = mC\,\Delta T$. For 1 g of Radium and 1 second,
0.0285 J = (1 g)(0.16 J/g C^o)(ΔT). In 1 second, ΔT = 0.18 C^o
In one hour, ΔT = (3600)(0.18 C^o) = about 640 C^o. (So it heats up very quickly!)

5. There are a million ways to do this problem. Here's a sample:

Assume the car gets 20 miles per gallon at a steady speed of 40 mi/hr. Consider a 20-mile trip; the time required is 1/2 hr or 1800 s, and one gallon of gasoline is consumed..

Estimate power requirement of headlights to be 100 W = 100 J/s. During the trip, total energy required by headlights is (100 J/s)(1800 s) = 1.8×10^5 J.

If the entire heat of combustion of gasoline (6×10^7 J/kg) could be converted to electrical energy, the *mass* of gasoline required to supply this energy would be

$\dfrac{1.8 \times 10^5\,\text{J}}{6 \times 10^7\,\text{J / kg}} = 3 \times 10^{-3}$ kg. But the efficiency of gasoline engines is only 20 to 25%, so a better guess would be about five times this, or 15×10^{-3} kg. What is the corresponding *volume*?

One gallon is about 4 liters. The density of water is about 1 kg/liter; the density of gasoline is about 3/4 this, so a gallon of gasoline has a mass of about 3 kg. Thus the *volume* of gasoline required to run the lights for 1/2 hr is about

$\dfrac{15 \times 10^{-3}\,\text{kg}}{3\,\text{kg / gal}} = 0.005\,\text{gal}$. This is roughly 1/2 % of the *total* gasoline consumption, so the result is to decrease the gas mileage from 20 mi/gal to 19.9 mi/gal.

Is this negligible, or is it yet another conspiracy on the part of the oil companies? Or both? You be the judge!

If the life of the car is 100,000 miles, it uses 5,000 gal of gasoline during its life, for a total fuel cost of around $7000. Leaving the headlights on all the time will increase this by about 25 gallons and cost the owner around $40. This amount buys you increased visibility during 2500 hours of daylight driving. Worth it? Probably; you decide!