51. a) \[
\frac{\partial y}{\partial x} = -Ak \sin(kx + \omega t) - Ak \sin(kx - \omega t).
\]

At \(x = 0\), \(\frac{\partial y}{\partial x} = -Ak \sin(\omega t) - Ak \sin(-\omega t) = 0\) because \(\sin(-\alpha) = -\sin(\alpha)\)

b) Use the cosine sum identities: \(\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta\):

\[
y = A \cos kx \cos \omega t - A \sin kx \sin \omega t + A \cos kx \cos \omega t + A \sin kx \sin \omega t
= 2A \cos kx \cos \omega t.
\]

Then redefine \(A\).

c) \(y(x, t) = A \cos kx \cos \omega t\). At \(x = L\), \(y(L, t)\) must be zero.

\[
\cos kL = 0, \quad kL = \left(n + \frac{1}{2}\right)\pi, \quad n = 0, 1, 2, 3, \ldots
\]

\[
\omega_n = c k_n = \frac{(n + \frac{1}{2})\pi c}{L}, \quad n = 0, 1, 2, 3, \ldots
\]

\[
y(x, t) = A \cos \left[\frac{(n + \frac{1}{2})\pi x}{L}\right] \cos \left[\frac{(n + \frac{1}{2})\pi ct}{L}\right], \quad n = 0, 1, 2, 3, \ldots
\]

The frequencies are \(odd\) multiples of the fundamental:

\[
\omega_o = \frac{\pi c}{2L}, \quad f_o = \frac{\omega_o}{2\pi} = \frac{c}{4L}; \quad \text{this is half as large as when there are nodes at both ends, and } n \quad \text{is the number of nodes (not counting the one at } x = 0).\]

52. a) For (38), solve the simultaneous equations, Eqs. (36) and (37), by any method. (Maple is easiest.)

For (39): \(k_1 = \omega/c_1, \quad k_2 = \omega/c_2\). Substitute into Eq. (38):

\[
B = \frac{\omega/c_1 - \omega/c_2}{\omega/c_1 + \omega/c_1} A = \frac{c_2 - c_1}{c_1 + c_2} A, \quad C = \frac{2\omega/c_1}{\omega/c_1 + \omega/c_2} A = \frac{2c_2}{c_1 + c_2} A.
\]

b) \(c_1 = \sqrt{\frac{F}{\mu_1}}, \quad c_2 = \sqrt{\frac{F}{\mu_2}}\). Substitute into Eq. (39):

\[
B = \frac{\sqrt{(F/\mu_2)} - \sqrt{(F/\mu_1)}}{\sqrt{(F/\mu_1)} + \sqrt{(F/\mu_2)}} A = \frac{\sqrt{\mu_1} - \sqrt{\mu_2}}{\sqrt{\mu_1} + \sqrt{\mu_2}} A,
\]

\[
C = \frac{2\sqrt{(F/\mu_2)}}{\sqrt{(F/\mu_1)} + \sqrt{(F/\mu_2)}} A = \frac{2\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}} A.
\]
53. a) Wave functions are the same as Eqs. (35). Boundary conditions at \( x = 0 \):
\[ y_+ = y_-, \quad A \cos(-\omega t) + B \cos(\omega t) = C \cos(-\omega t), \]
and
\[ A + B = C. \]
\[ F_1 \frac{\partial y_+}{\partial x} = F_2 \frac{\partial y_+}{\partial x}, \quad F_1[-Ak_1 \sin(-\omega t) - Bk_1 \sin(\omega t)] = -F_2 Ck_2 \sin(-\omega t), \]
\[ F_1 k_1 A - F_1 k_2 B = F_2 k_2 C. \]
Solve simultaneous equations for \( B \) and \( C \), then use
\[ \frac{F_1}{c_1} = \frac{F_1}{\sqrt{F_1/\mu}} = \sqrt{\mu F_1}; \]
\[ B = \frac{F_1 k_1 - F_2 k_2}{F_1 k_1 + F_2 k_2} A = \frac{F_1/c_1 - F_2/c_2}{F_1/c_1 + F_2/c_2} A = \frac{\sqrt{F_1} - \sqrt{F_2}}{\sqrt{F_1} + \sqrt{F_2}} A, \]
\[ C = \frac{2F_1 k_1}{F_1 k_1 + F_2 k_2} A = \frac{2F_1 c_1}{F_1/c_1 + F_2/c_2} A = \frac{2\sqrt{F_1}}{\sqrt{F_1} + \sqrt{F_2}} A. \]
Note that if both \( F \) and \( \mu \) are different on the two sides, then
\[ B = \frac{\sqrt{\mu_1 F_1} - \sqrt{\mu_2 F_2}}{\sqrt{\mu_1 F_1} + \sqrt{\mu_2 F_2}} A \quad \text{and} \quad C = \frac{2\sqrt{\mu_1 F_1}}{\sqrt{\mu_1 F_1} + \sqrt{\mu_2 F_2}} A. \]
The characteristic impedance of each side is \( Z = \sqrt{\mu F} \). If \( Z_1 = Z_2 \), then there is no reflected wave.

54. \[ y(x, t) = A \cos(kx + \omega t), \quad v_y = -A \omega \sin(kx + \omega t), \]
\[ F_y = -F \frac{\partial y}{\partial x} = FAk \sin(kx + \omega t). \]
At \( x = 0 \), \( v_y = -A \omega \sin \omega t, \quad F_y = FAk \sin \omega t, \)
\[ \frac{F_y}{v_y} = -\frac{FAk \sin \omega t}{A \omega \sin \omega t} = -\frac{Fk}{\omega} = -\frac{F}{c} = \frac{F}{\sqrt{F/\mu}} = \sqrt{\mu F}. \]
So \( F_y = -bv_y \), where \( b = \sqrt{\mu F} \). What we need is a shock absorber with a damping constant equal to \( b \).