33-231 Physical Analysis Fall 2003

Problems: Set 6 (due Wednesday, October 8, 2003)

25. A grandfather clock has a pendulum 1.000 m long, with mass $m = 0.200$ kg and period 2.000 s. The amplitude of the pendulum swing is 0.100 rad. The clock is powered by the work done by gravity on a dropping 0.500-kg weight ($M = 0.500$ kg) that drops 0.800 m per day.

    a) Determine the value of g at the location of the clock.

    b) Determine the Q of the clock.

    c) How long would the clock run if it were powered by a battery with an energy capacity of 5.00 J? What kind of battery would have this capacity?

26. A critically damped harmonic oscillator has initial position x_o and initial velocity v_o (at time $t = 0$). A sinusoidal driving force $F_o \cos \omega t$ is applied.

    a) Derive an expression for $x(t)$ that is consistent with these conditions. Note that this is not the same situation as Problem 22(a), and the answer is different. However, in the particular case where there is *no* driving force (i.e., $A' = 0$) your result should reduce to that in Problem 22(a). Check to see that this actually happens.

    b) In part (a), the general solution consists of a free-oscillation or "transient" part that depends on the initial conditions, and a forced-oscillation or "steady-state" part that depends on the amplitude and frequency of the driving force. Show that if the system is given exactly the right initial conditions (i.e., particular values of x_o and v_o) the transient part vanishes. Derive expressions for the values of x_o and v_o needed for this to occur.

27. A tuning fork has a free-oscillation frequency $f = 440$ Hz. We represent it as a lightly damped harmonic oscillator with mass $m = 0.010$ kg and $Q = 4400$.

    a) If a sinusoidal driving force is applied, with a frequency of 440 Hz, what must be the force amplitude F_o if the forced-oscillation amplitude is 2.00 mm?

    b) The frequency of the driving force is now changed to 441 Hz, with the same force amplitude as in (a). What is the amplitude of the oscillation now?

    c) Now we increase the damping to change Q to 440. What are the answers to (a) and (b) now?

    d) What value of Q would be needed in order to change the free-oscillation frequency to 439.9 Hz?

28. In Problem 21, a particle with mass m moves along the x axis under the action of a force $F(x)$ given by $F = -k\left(\frac{x}{a}\right)^{19}$.

Now we want to add a velocity-proportional damping force $F = -bv = -b\dot{x}$, perhaps due to air resistance.

a) Using the numerical values in Problem 21(c), write the appropriate differential equation, and obtain a numerical solution. Start with $x_o = 0$, $v_o = 1$, and $b = 0.1$. Later you can experiment with other values. Plot a graph of $x(t)$, taking a large enough range for t to include several cycles. Comment on any conspicuous or unexpected features of the graph.

b) Make a phase plot for the solution in (a). Again include several cycles. Discuss why the long, nearly straight portions of the curve should be expected.

c) (optional) Experiment with larger values of b (e.g., 0.2 or 0.3). You may find that the mass appears to come to rest at a point other than the equilibrium position. Try to understand what is happening.

29. A "super-ball" bounces back and forth between two perfectly rigid walls. There is air resistance (due to turbulent flow) proportional to v^2. We want to construct a mathematical model of the situation and use it to analyze the motion. Some of the analysis may resemble that in Problem 28. Make up your own problem; state the problem clearly and solve it. Here are a few thoughts:

a) In Problem 21 we noted the similarity of the force function to a situation with rigid walls. Can we do the same thing here? If x^{19} is good, would x^{99} be even better? Greater precision? Would it slow down the calculation too much?

b) You have to do something to make sure the resisting force has the right sign in your differential equation. The Maple `signum` or `abs` function may be useful; read the Maple help files for details. Do you want plots of $x(t)$, phase plots, predictions of the period of the motion, or what?