33-231 Physical Analysis Fall 2003

Problems: Set 1 (due Wednesday, September 3, 2003)

Note: Before you begin work on this set of problems, please read carefully pages 3 and 4 of the Course Outline, titled "*About Problems*."

1. Introduction to Maple: Find a computer and work through all the examples in **Introduction to Maple,** starting at the bottom of page 1-3 (Arithmetic) and going through to the end of page 1-8. Print the Maple output for pages 1-6 through 1-8 and hand it in.

2. Consider again the canoe problem we discussed in class on Wednesday (August 27). Suppose the resisting force of the water is proportional not to v, but to v^2. Again let the proportionality constant be b. Carry out an analysis parallel to our discussion in class.

   a) Write the differential equation for v. Solve this equation by separation of variables, using the initial condition that the value of v at time $t = 0$ is v_0.

   b) Separate variables again and integrate, using the initial condition that the position at time $t = 0$ is x_0. You may want to use Maple to evaluate the integral, although it can also be done easily by a simple substitution.

   c) Derive a general expression for the *time* at which the speed has decreased from its initial value v_o to v_o/e. Also derive a general expression for the total *distance* the canoe travels during this time.

   d) Suppose that the resisting force has magnitude 20 N when the speed is 1.0 m/s. Find the numerical value of b; be sure to find the correct units for b. Are these units consistent with your result for the distance in part (c)?

   e) Assuming $m = 100$ kg and $v_o = 2.0$ m/s obtain a numerical value for the approximate time at which the canoe's speed has decreased to $1/e$ of its initial value. Use the Maple `solve` or `fsolve` command if you like. Also obtain a numerical value for the distance the canoe moves during this time.

3. A specimen of a radioactive element contains N radioactive nuclei at time t. The probability that an individual nucleus will decay in a time dt is λdt, where λ is a constant characteristic of the element, called the *decay constant*. Thus the total number of nuclei that decay in time dt is $\lambda N\, dt$, and the resulting change in N is given by

$$dN = -\lambda N\, dt, \qquad \text{or} \qquad \frac{dN}{dt} = -l\ N.$$

a) If the number of nuclei at time $t = 0$ is N_0, derive an expression for the number N of nuclei at time t.

b) The *half-life* of the element, denoted by $t_{1/2}$, is defined as the time for the number of radioactive nuclei to decrease to one-half the original number. Derive an expression for $t_{1/2,}$ in terms of λ and numerical constants.

c) The *decay time* of the element, denoted by τ, is defined as the time for the number of radioactive nuclei to decrease to $1/e$ of the original number. Derive an expression for τ, in terms of λ and numerical constants. (In reference to the decay of fundamental particles, τ is also called the *lifetime* of the particle.)

4. This problem continues the analysis of radioactive decay begun in Problem 3.

a) The *activity* of a particular specimen of material is defined as the number of disintegrations per unit time. The SI unit of activity is the *bequerel*, abbreviated Bq. 1 Bq = 1 decay/s. Show that the activity is given by $N\lambda$.

b) The element Radium (^{226}Ra) decays to Radon (^{222}Rn) by emitting an alpha particle. The half-life is 1600 years. Find the activity of one gram of Radium. You will first have to find the number of atoms, using the atomic mass and Avogadro's number. Then find the decay constant λ from the half-life (taking care to convert units appropriately). Finally, find the activity from N and λ.

The activity of one gram of ^{226}Ra was formerly used as a *unit* of activity; this unit was called *one curie* (abbreviated Ci). What is the conversion factor from bequerels to curies?

c) When a Radium (^{226}Ra) nucleus decays, the total kinetic energy of the resulting α particle and Radon (^{222}Rn) nucleus is 4.87 MeV. Find the corresponding *power* (energy per unit time) emitted by one gram of Radium. Express your result in watts.

d) If all the energy evolved in the decay of ^{226}Ra to ^{222}Rn remains in the material as heat, estimate the rise in temperature in one hour. The specific heat capacity of Radium is about 0.16 J/g C^o.

5. Some new cars are wired so that the headlights are always on, even in broad daylight, whenever the engine is running. A question arises as to whether this use of energy increases the gasoline consumption appreciably.

   Make reasonable guesses as to the magnitudes of the various quantities involved, and obtain an answer to this question. You will of course have to define what "appreciably" means. The heat of combustion of gasoline is approximately 6×10^7 J/kg. but not all this energy is converted into useful work. (Why not?)