33-131 Exam 3, Monday 2000 Nov. 20

Name____Section______

- **Read all problems carefully before attempting to solve them.**
- **Your work must be legible, and the organization must be clear.**
- **Correct answers without adequate explanation will be counted wrong.**
- **Incorrect explanations mixed in with correct explanations will be counted wrong.**
- **Make explanations complete but brief. Do not write a lot of prose.**
- **Include diagrams!**
- **Show what goes into a calculation, not just the final number:** $\frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
- **Give physical units with your results.**

If you cannot do some portion of a problem, invent a symbol for the quantity you can't calculate (explain that you're doing this), and do the rest of the problem.

Problem Score

1 (40 pts):________

2 (25 pts):________

3 (25 pts):________

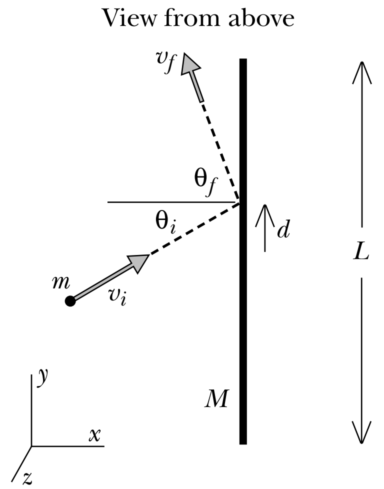
4 (10 pts):________

5 (3 pts):_________

Total: ________

Problem 1 (40 pts)

A stick of mass M and length L is lying on ice. A small mass m traveling at high speed v_i strikes the stick a distance d from the center and bounces off with speed v_f as shown in the diagram, which is a top view of the situation. The magnitudes of the initial and final angles to the x axis of the small mass's velocity are θ_i and θ_f. All of the symbols in the diagram represent positive numbers. See the formula page for additional information.



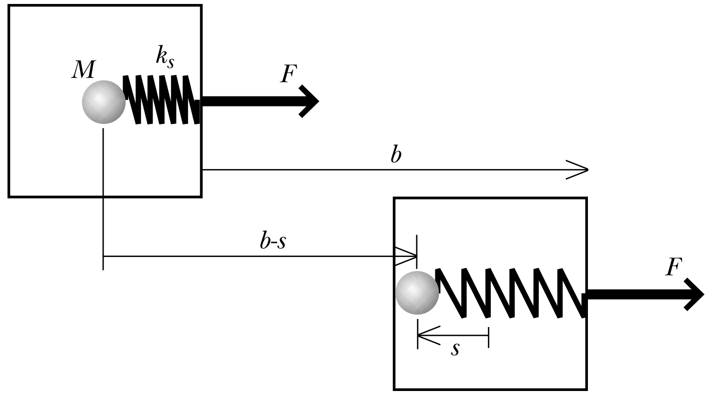
(a 15 pts) Afterwards, what are the velocity components v_x and v_y of the center of the stick? Explain briefly.

(b, 15 pts) Afterwards, what is the magnitude and direction of the angular velocity $\vec{\omega}$ of the stick?

(c, 10 pts) What is the increase in thermal energy of the objects? You can leave your expression in terms of the initial quantities and v_x, v_y, and ω.

Problem 2 (25 pts)

A thin box in outer space contains a large ball of clay of mass M, connected to an initially relaxed spring of stiffness k_s. The mass of the box is negligible compared to M. The apparatus is initally at rest. Then a force of constant magnitude F is applied to the box. When the box has moved a distance b, the clay makes contact with the left side of the box and sticks there, with the spring stretched an amount s. See the diagram for distances.



(a, 10 pts) Immediately after the clay sticks to the box, how fast is the box moving?

(b, 15 pts) What is the increase in thermal energy of the clay?

Problem 3 (25 pts)

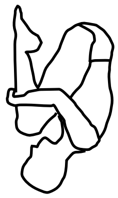


Figure 1. Tucked position.

A diver dives from a platform 10 meters high. When he leaves the platform, he tucks tightly (figure 1) and performs three complete revolutions in the air before straightening out and entering the water with his body fully extended (figure 2). He is in the air for a total time of 1.4 seconds. What is his angular speed ω just as he enters the water? Give a numerical answer.

Be explicit about the details of your model. Calculations without (brief) explanations will be counted wrong. You will need to estimate some quantities. Some formulas on the formula sheet may be helpful.

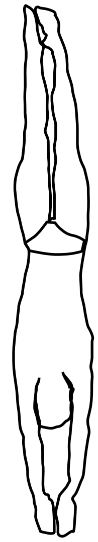


Figure 2. Entering water.

Problem 4 (10 pts)

When the Rutherford group started their experiment, they expected to find very low-mass electrons embedded in a low-density pudding of positive charge. Explain why Rutherford was surprised when alpha particles bounced back from the gold foil, and what this observation led him to propose.

Problem 5 (3 pt bonus question)

If you get a total score greater than 100, we round down to 100; since this problem is only worth 3 bonus points, don't attempt it unless you have finished all the other problems and checked your work.

Explain what aspects of the experiment Rutherford used to show that an ordinary *electric* interaction was sufficient to model the interaction of alpha particles with the gold foil.

Fundamental Principles

The momentum principle, the energy principle, and the angular momentum principle.

Specific results and data

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{relative to the cm}} = K_{\text{trans}} + K_{\text{rotation}} + K_{\text{vibration}} + K_{\text{explosion}}$$

$K_{\text{trans}} \approx \dfrac{P_{\text{total}}^2}{2M} = \frac{1}{2}Mv_{\text{cm}}^2$ $\qquad$ $K_{\text{rotation}} = \dfrac{L_{\text{rot}}^2}{2I} = \frac{1}{2}I\omega^2$

$\vec{\tau}_A = \vec{r}_A \times \vec{F}$ $\qquad$ $\vec{L}_A = \vec{L}_{\text{trans},A} + \vec{L}_{\text{rot}} = (\vec{R}_{\text{cm,A}} \times \vec{P}_{\text{total}}) + \vec{L}_{\text{rot}}$ $\qquad$ $\vec{L}_{\text{rot}} = I\vec{\omega}$

$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$ $\qquad$ $\left|\dfrac{d\vec{X}}{dt}\right| = \omega X$ if $\vec{X}$ is a rotating vector

$E^2 = (Pc)^2 + (mc^2)^2$ $\qquad$ $\vec{R}_{\text{cm}} \equiv \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$

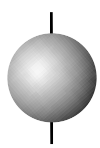
$L_z = N\hbar$ $N = (0, 1, 2, \ldots)$ or $(1/2, 3/2, 5/2, \ldots)$ $\qquad$ $L^2 = l(l+1)\hbar^2$ $l = (0, 1, 2, \ldots)$ or $(1/2, 3/2, 5/2, \ldots)$

$m_{\text{proton}} \approx m_{\text{neutron}} \approx m_{\text{hydrogen atom}} = \dfrac{1\times10^{-3}\ \text{kg/mole}}{6\times10^{23}\ \text{atoms/mole}} = 1.7\times10^{-27}\ \text{kg}$ $\qquad$ $m_{\text{electron}} = 9\times10^{-31}\ \text{kg}$

$G = 6.7\times10^{-11}\ \text{N·m}^2/\text{kg}^2$ $\qquad$ $g = 9.8\ \text{N/kg}$ $\qquad$ $h = 6.6\times10^{-34}\ \text{J·s}$ $\qquad$ $\hbar = 1.05\times10^{-34}\ \text{J·s}$ $\qquad$ $1\ \text{eV} = 1.6\times10^{-19}\ \text{J}$

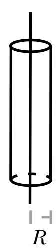
$\dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N·m}^2/\text{C}^2$ $\qquad$ $c = 3\times10^8\ \text{m/s}$ $\qquad$ $E_N = -(13.6\ \text{eV})/N^2$ $\qquad$ $R \approx (1.3\times10^{-15}\ \text{m})A^{1/3}$

Sphere



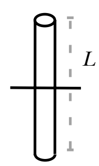
$I = \frac{2}{5}MR^2$

Cylinder or disk



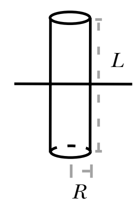
$I = \frac{1}{2}MR^2$

Thin rod (about axis shown)



$I = \frac{1}{12}ML^2$

Solid cylinder (about axis shown)



$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$