ME 24-731
Conduction and Radiation Heat Transfer

Assignment No: 2
Due Date: February 16, 2000
Spring 2000
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1. Consider a triangular fin as shown in Figure 1. The fin extends infinitely into the page, and loses heat to the surroundings. This heat loss may be modeled as a convective heat transfer to a surroundings temperature of T_∞ through a heat transfer coefficient h. Show that the fin heat transfer equation for this geometry is given by:

$$\frac{\partial}{\partial x}\left(x\frac{\partial \theta}{\partial x}\right) - m^2\theta = 0 \tag{1}$$

where

$$m^2 = \frac{2hL}{kb}$$

and $\theta = T - T_b$.
By using the transformation

$$z = 2mx^{\frac{1}{2}}$$

show that Equation 1 can be transformed into the *modified Bessel's equation*

$$z^2\frac{\partial^2\theta}{\partial z^2} + z\frac{\partial \theta}{\partial z} - \theta z^2 = 0$$

Though you are not required to find the solution, it may interest you to know that it can be written in terms of modified Bessel's functions as:

$$\theta = C_1 I_0(z) + C_2 K_0(z)$$

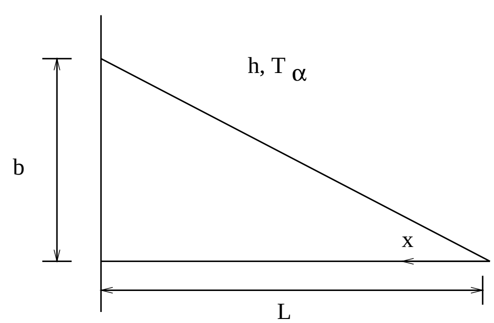

Figure 1: Fin Geometry for Problem 1

2. Let us explore to solution derived in class for steady two-dimensional conduction in the rectangular domain shown in Figure 2. For $T_1 = 200\ K$ and $T_2 = 500\ K$, $L = 1\ m$ and $W = 2\ m$

   (a) Find how many terms of the series must be summed to find the temperature at (0.5,1.0) to an accuracy of 1%.

   (b) Plot the temperatures on lines x=0.25m, x=0.5m, and x=0.75m.

   (c) Show that the maximum and minimum temperatures lie on the boundaries of the domain.

   (d) Find an expression for the heat flux on the top boundary, $q''(x, 2.0)$.

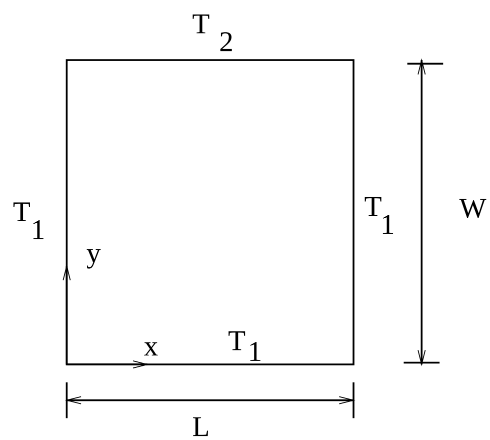

Figure 2: Conduction in a Rectangular Domain (Problem 2)

3. Let us explore the separation of variables technique which we are using extensively in class to study conduction problems. Consider steady 2-D conduction with constant properties. For each of the cases listed below, determine whether the separation of variables technique will work or not by repeating the derivation done in class.

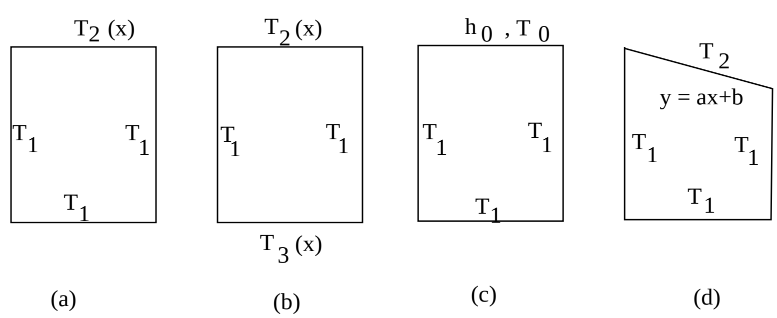

Figure 3: Boundary Conditions for Problem 3