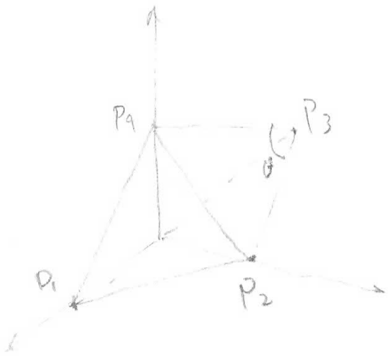
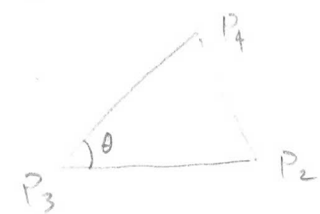


$P_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $P_2 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ $P_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ $P_4 = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

(1) Find distance $\overline{P_1P_3}$

$\sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

(2) Find $\angle P_3 P_2 P_4$



$\overline{P_3P_4} = \sqrt{5}$ $\overline{P_2P_4} = \sqrt{12} = 2\sqrt{3}$

$\overline{P_2P_3} = \sqrt{5}$

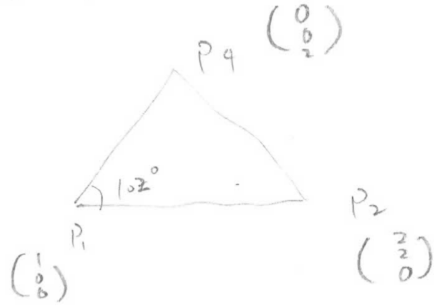
$(\underbrace{\overline{P_4P_2}}_{=12})^2 = (\underbrace{\overline{P_2P_3}}_{=5})^2 + (\underbrace{\overline{P_3P_4}}_{=5})^2 - 2 \cdot \overline{P_2P_3} \cdot \overline{P_3P_4} \cos\theta$

$10 \cos\theta = -2$

$\cos\theta = -0.2$

$\theta = 102°$

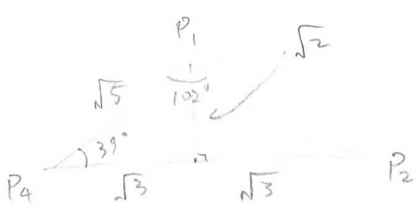
(3) Find area $\Delta P_1 P_2 P_4$



$\overline{P_1P_2} = \sqrt{5}$

$\overline{P_1P_4} = \sqrt{5}$

$\overline{P_4P_2} = \sqrt{12}$



area $= \sqrt{6}$

(1) $\overline{P_1P_3} = \left\| \begin{pmatrix}0\\2\\1\end{pmatrix} - \begin{pmatrix}1\\0\\0\end{pmatrix} \right\| = \left\|\begin{pmatrix}-1\\2\\1\end{pmatrix}\right\| = \sqrt{(-1)^2+2^2+1^2} = \sqrt{6}$

(2).

$$\theta = \arccos \frac{\overline{P_2P_3}\cdot\overline{P_4P_2}}{|\overline{P_2P_3}|\cdot|\overline{P_4P_2}|} = \arccos \frac{\begin{pmatrix}-2\\0\\1\end{pmatrix}\cdot\begin{pmatrix}-2\\-2\\2\end{pmatrix}}{\sqrt{(-2)^2+1^2}\cdot\sqrt{(-2)^2+2^2+2^2}} = \arccos\frac{4+2}{\sqrt{5}\cdot\sqrt{12}} = \arccos\frac{\sqrt{15}}{5}$$

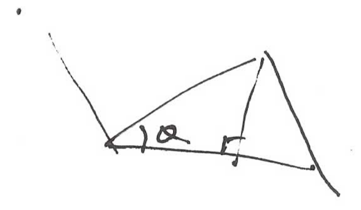
(3) $S_{\triangle P_1P_2P_4} = |\overline{P_4P_2}|\cdot|\overline{P_1P_2}|\cdot\sin\angle P_4P_2P_1$

(1) $\overline{P_1P_3} = \sqrt{1^2+2^2+1^2} = \sqrt{6}$

(2) $\cos\angle P_3P_2P_4 = \dfrac{\overrightarrow{P_2P_4}\cdot\overrightarrow{P_2P_3}}{\overline{P_2P_4}\ \overline{P_2P_3}} = \dfrac{\begin{pmatrix}-2\\-2\\2\end{pmatrix}\cdot\begin{pmatrix}-2\\0\\1\end{pmatrix}}{\sqrt{12}\ \sqrt{5}} = \dfrac{6}{\sqrt{60}} = \sqrt{\dfrac{3}{5}}.$

$\angle P_3P_2P_4 = \operatorname{acos}\sqrt{\dfrac{3}{5}}$

(3) $\overrightarrow{P_1P_4} = \begin{pmatrix}-1\\0\\2\end{pmatrix} \quad \overrightarrow{P_1P_2} = \begin{pmatrix}1\\2\\0\end{pmatrix}$



$\cos\angle P_4P_1P_2 = \dfrac{\overrightarrow{P_1P_4}\cdot\overrightarrow{P_1P_2}}{\overline{P_1P_4}\ \overline{P_1P_2}} = \dfrac{-1}{5}$

$\Rightarrow \sin\angle P_4P_1P_2 = \sqrt{1-(\cos\angle P_4P_1P_2)^2} = \dfrac{2\sqrt{6}}{5}$

$\Rightarrow \triangle P_1P_2P_4 = \dfrac{1}{2}\cdot\overline{P_1P_2}\cdot\overline{P_1P_4}\cdot\sin\angle P_4P_1P_2$

$= \dfrac{1}{2}\times\sqrt{5}\times\sqrt{5}\times\dfrac{2\sqrt{6}}{5}$

$= \sqrt{6}$

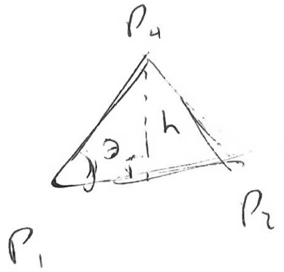
1. $\overline{P_1P_3} = \sqrt{(1-0)^2+(0-2)^2+(0-1)^2} = \sqrt{6}$

2. $\angle P_2P_3P_4$

$$\overline{P_2P_3} = \sqrt{(2-0)^2+(2-2)^2+(0-1)^2} = \sqrt{5}$$

$$\overline{P_2P_4} = \sqrt{(2-0)^2+(2-0)^2+(0-2)^2} = \sqrt{12} = 2\sqrt{3}$$

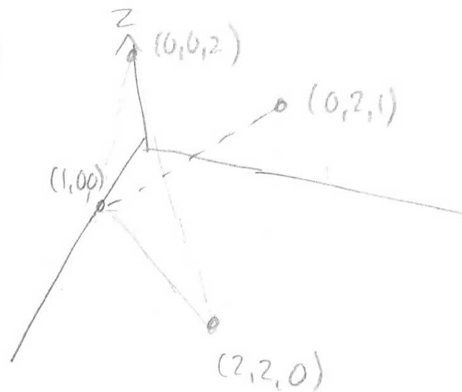
3.



$$\frac{h}{\overline{P_1P_4}} = \sin\theta \Rightarrow h = \overline{P_1P_4}\cdot\sin\theta$$

$$A = \overline{P_1P_2}\cdot h/2 = \overline{P_1P_2}\cdot\overline{P_1P_4}\,\frac{\sin\theta}{2}$$

1)



$$D = \sqrt{(x-x_1)^2 + (y-y_2)^2 + (z-z_2)^2}$$

$$D = \sqrt{(1)^2 + (2)^2 + (1)^2} = \sqrt{6}$$

2) angle b/w P_3, P_2, P_4

$$\theta = \tan^{-1}\left(\frac{\sqrt{5}}{1}\right)$$

3) area P_1, P_2, P_4

$$\text{area } \Delta = \frac{l \cdot h}{2}$$

$$\overline{P_1 P_4} = h = \sqrt{1^2 + 2^2} = \sqrt{5}$$

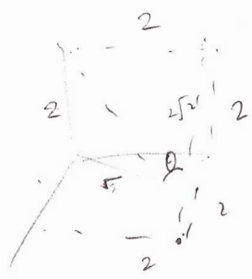
$$\overline{P_1 P_2} = l = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\text{area} = \frac{(\sqrt{5})(\sqrt{5})}{2} = \frac{5}{2}$$

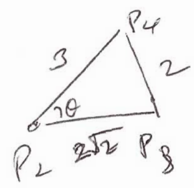
(1)

$\sqrt{5+1} = \sqrt{6} = \overline{P_1P_3}$

(2)

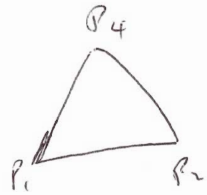


$\overline{P_4P_2} = 3$



$= \cos^{-1}\left(\frac{\sqrt{50}}{5}\right)$

(3)



$\sqrt{6}$

(1) $\overline{P_1P_3} = \sqrt{(P_{3x} - P_{1x})^2 + (P_{3y} - P_{1y})^2 + (P_{3z} - P_{1z})^2}$

$= \sqrt{(-1)^2 + (2)^2 + (1)^2}$

$\boxed{\overline{P_1P_3} = \sqrt{6}}$

(2) $\angle P_3P_2P_4 = \quad c^2 = a^2 + b^2 - 2ab\cos C$

(3) Area of $\Delta P_1P_2P_4 = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{1}{2}(a+b+c)$

$\overline{P_1P_2} = \sqrt{(2-1)^2 + (2)^2} = \sqrt{(1)^2 + 4}$

$= \sqrt{5}$

$\overline{P_1P_4} = \sqrt{(-1)^2 + (2)^2}$

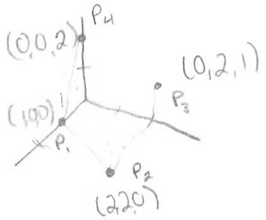
$= \sqrt{5}$

$\overline{P_2P_4} = \sqrt{(2)^2 + (2)^2 + (2)^2}$

$= \sqrt{12} = 2\sqrt{3}$

$\frac{1}{2}(2\sqrt{5} + 2\sqrt{3})$

$= (\sqrt{5} + \sqrt{3})$



1) $\overline{P_1P_3} = (2^2+1^2+1^2)^{1/2} = \boxed{\sqrt{6}}$

2) $\angle P_3P_2P_4 : \overline{P_3P_4}^2 = \overline{P_2P_3}^2 + \overline{P_2P_4}^2 - 2(\overline{P_2P_3})(\overline{P_2P_4})\cos\theta$

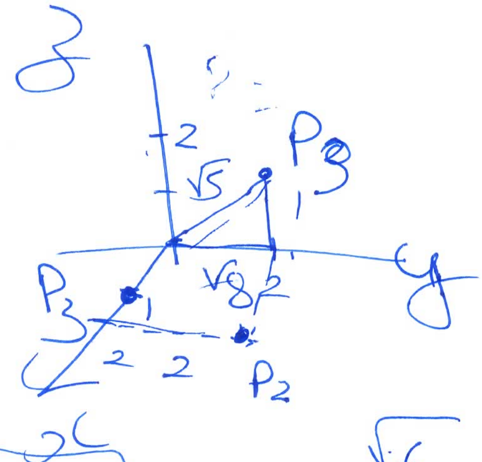
3) $\triangle P_1P_2P_4 = \frac{1}{2}\overline{P_1P_2}(\overline{P_1P_4})$

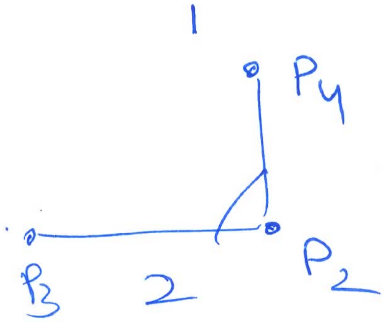
$= \frac{1}{2}(1^2+2^2)^{1/2}(1^2+2^2)^{1/2}$

$= \frac{1}{2}(\sqrt{5})(\sqrt{5})$

$= \boxed{\frac{5}{2}}$

1) →



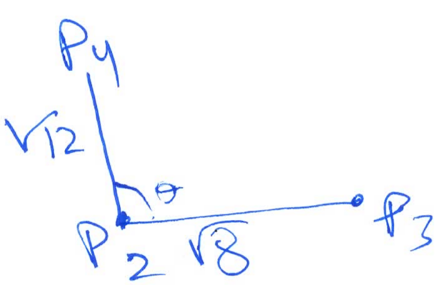


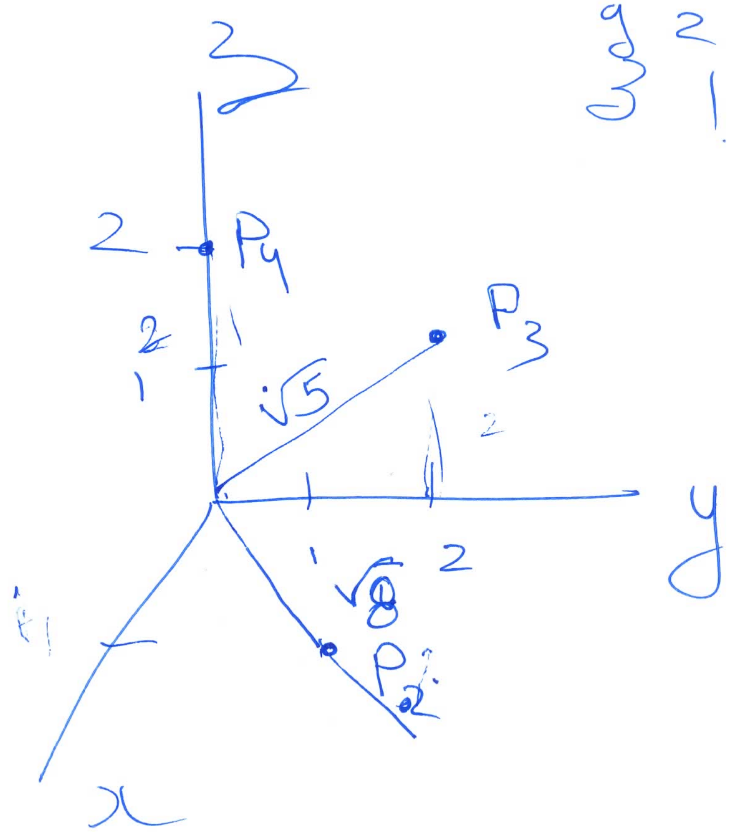
$\boxed{P_1 P_3 = \sqrt{6}}$

$\sqrt{6}$

P_3	
x	0
y	2
z	1

2) →





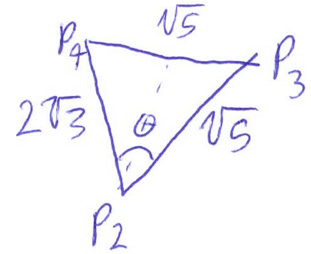
$8+4$

$\sqrt{2}$

$P_0 = P_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, P_2 = \begin{pmatrix}2\\2\\0\end{pmatrix}, P_3 \begin{pmatrix}0\\2\\1\end{pmatrix}, P_4 = \begin{pmatrix}0\\0\\2\end{pmatrix}$

① $\sqrt{(1-0)^2+(0-2)^2+(0-1)^2} = \sqrt{1+4+1} = \sqrt{6}$ P_1P_3 distance

②



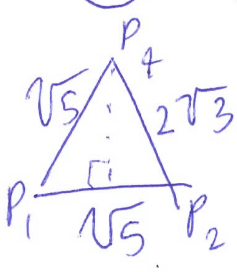
P_2P_3 $\sqrt{(2-0)^2+(2-2)^2+(0-1)^2} = \sqrt{4+1} = \sqrt{5}$

P_2P_4 $\sqrt{(2-0)^2+(2-0)^2+(0-2)^2} = \sqrt{4+4+4} = \sqrt{12}$
$= 2\sqrt{3}$

P_3P_4 $\sqrt{0+(2-0)^2+(1-2)^2} = \sqrt{4+1} = \sqrt{5}$

there's formula to calculate ~~the~~ this once you have all sides

③ P_1P_2 $\sqrt{(1-2)^2+(0-2)^2+(0-0)^2} = \sqrt{1+4} = \sqrt{5}$



P_1P_4 $\sqrt{(1-0)^2+(0-0)^2+(0-2)^2} = \sqrt{1+4} = \sqrt{5}$

$A = \frac{1}{2}bh$

$\frac{\sqrt5}{2}$

$A = \frac{1}{2}(\sqrt{5})$