

Quiz #6 Solutions

known: a, b, ω

Independent variable: θ

Dependent variables: x, ϕ

1. Find ϕ, x in terms of θ :

$$\frac{a}{\sin\theta} = \frac{b}{\sin\phi} \Rightarrow \sin\phi = \frac{b}{a}\sin\theta, \quad \boxed{\phi = \sin^{-1}\left(\frac{b}{a}\sin\theta\right)}$$

$$x = b\cos\theta + a\cos\phi \Rightarrow \boxed{x = b\cos\theta + a\sqrt{1 - \left(\frac{b}{a}\sin\theta\right)^2}}$$

2. Find v, a :

$$v = \frac{dx}{dt} = -(b\sin\theta)\frac{d\theta}{dt} + \frac{a}{2}\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-1/2} \left(-2\frac{b}{a}\sin\theta\right)\left(\frac{b}{a}\cos\theta\right)\frac{d\theta}{dt}$$

$$\boxed{v = -b\omega\sin\theta - \frac{b^2\omega}{2a}\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-1/2} \cdot \sin 2\theta}$$

$$a = \frac{dv}{dt} = -b\omega^2\cos\theta - \frac{b^2\omega}{2a}\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-1/2} \cdot \cos 2\theta \cdot 2\omega$$

$$+ \sin 2\theta \cdot \frac{b^2\omega}{2a} \left(-\frac{1}{2}\right) \left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-3/2} \left(-2\left(\frac{b}{a}\sin\theta\right)\right) \frac{b}{a} (\cos\theta)\omega$$

$$\boxed{a = -b\omega^2\cos\theta - \frac{b^2\omega^2}{a}(\cos 2\theta)\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-1/2} + \sin 2\theta \frac{b^4\omega^2}{4a^3}\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-3/2}}$$

3. Find $\dot{\phi}, \ddot{\phi}$:

differentiate $\sin\phi = \frac{b}{a}\sin\theta$, get $\cos\phi \cdot \dot{\phi} = \frac{b}{a}\cos\theta\omega$

$$\boxed{\dot{\phi} = \frac{b\omega}{a}\cos\theta\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-1/2}}$$

$$\ddot{\phi} = \frac{d\dot{\phi}}{dt} = \frac{b\omega}{a}\cos\theta \cdot \left(-\frac{1}{2}\right)\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-3/2} \left(-2\frac{b}{a}\sin\theta\right)\left(\frac{b}{a}\cos\theta\right)\omega$$

$$+ \frac{b\omega}{a}\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-1/2} \cdot (-\sin\theta) \cdot \omega$$

$$\boxed{\ddot{\phi} = \frac{b^3\omega^2}{a^3}\cos^2\theta\sin\theta\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-3/2} - \frac{b\omega^2}{a}\sin\theta\left(1 - \left(\frac{b}{a}\sin\theta\right)^2\right)^{-1/2}}$$