

Quiz #5 Solutions

1. Just before collision, A has velocity of $\sqrt{2gh}$

Conservation of Momentum:

$$m_1 \sqrt{2gh} = (m_1 + m_2) v_0 \Rightarrow v_0 = \frac{m_1}{m_1 + m_2} \sqrt{2gh}$$

2. Mech. Energy of Spring initially: $\frac{1}{2} k y_0^2$ ($y_0 = \frac{m_2 g}{k}$)

$$\text{Mech. Energy before collision: } \frac{1}{2} m_1 (\sqrt{2gh})^2 + \frac{1}{2} k y_0^2$$

$$\text{Mech. Energy after collision: } \frac{1}{2} (m_1 + m_2) v_0^2 + \frac{1}{2} k y_0^2$$

So Mech. Energy is not conserved.

Loss of energy during collision:

$$\Delta E = \frac{1}{2} m_1 (\sqrt{2gh})^2 - \frac{1}{2} (m_1 + m_2) v_0^2 = \frac{m_1 m_2}{m_1 + m_2} gh$$

3. Conservation of energy after the collision:

$$\frac{1}{2} (m_1 + m_2) v_0^2 + \frac{1}{2} k y_0^2 = - (m_1 + m_2) g y_{\max} + \frac{1}{2} k (y_0 + y_{\max})^2$$

$$y_{\max}^2 - \frac{2m_1 g}{k} y_{\max} - \frac{2m_1^2 g h}{k(m_1 + m_2)} = 0 \quad (y_{\max} > 0)$$

$$\Rightarrow y_{\max} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{m_1 g}{k} + \sqrt{\frac{m_1^2 g^2}{k^2} + \frac{2m_1^2 g h}{k(m_1 + m_2)}}$$

$$\Rightarrow y_{\max} = \frac{m_1 g}{k} \left(1 + \sqrt{1 + \frac{2kh}{(m_1 + m_2)g}} \right)$$

