

24-311 NUMERICAL METHODS Fall 03

Carnegie Mellon University

PROBLEM SET 6

Issued: 10/03/2003
Due: 10/10/2003 Friday 1:00PM @ HH B127
Weight: 4% of total grade

PS6-1 Gauss-Seidel

Consider the following three linear equations:

$$15x + y + 17z = 10$$

$$24x + y = 1$$

$$x + 5y + z = 5$$

Use the Gauss-Seidel method to solve for x , y , and z . Run the Gauss-Seidel iteration three times. Use the initial guess of $x = y = z = 1$. Show all the intermediate steps for full credit.

PS6-2 LU Decomposition and Inverse Matrix

(1) Solve the following set of equations with LU decomposition with partial pivoting. Show all the intermediate steps.

$$x_1 + x_2 + 2x_3 = 9$$

$$x_1 + 3x_2 + x_3 = 10$$

$$4x_1 + 2x_2 + 4x_3 = 20$$

(2) Using the LU decomposition, find the inverse matrix of the 3×3 matrix consisting the coefficients of the above three linear equations decomposition (the method was covered in the 10/2 lecture).

PS6-3 The Golden-Section Search

Given the function

$$f(x) = -1.5x^6 - 2x^4 + 12x$$

- (1) Plot the function using Mathcad.
- (2) Prove that the function is convex for all values of x except at $x = 0$ using analytical methods.
- (3) Solve for the value of x that maximizes $f(x)$ using the golden-section search. Employ initial guesses of $x_l = 0$ and $x_u = 2$ and perform 4 iterations. Show your iteration results in the following format:

i	x_l	x_2	x_1	x_u
1	0	???	???	2
2	???	???	???	???
3	???	???	???	???
4	???	???	???	???

- (4) Calculate the maximum value of the function using the result of PS6-3 (3).

PS6-4 The Quadratic Interpolation

- (1) Solve for the value of x that maximizes $f(x)$, the same function used in PS6-3, using quadratic interpolation. Employ initial guesses of $x_0 = 0$, $x_1 = 1$ and $x_2 = 2$ and perform 4 iterations. Show your iteration results in the following format:

i	x_0	x_1	x_2	x_3
1	0	1	2	???
2	???	???	???	???
3	???	???	???	???
4	???	???	???	???

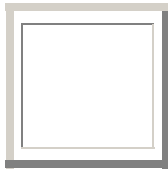
- (2) Calculate the maximum value of the function using the result of PS6-4 (1).

PS6-5 Using Newton's method, find the maximum of

$$f(x) = x^4 - x^3 - 1$$

with an initial guess of $x_0 = 2$. Run three iterations. You need to show all the intermediate steps for full credit.

PS6



The first letter of
your LAST name

_____ First Name

_____ Last Name

PS6-1 (20 pts)	PS6-2 (20 pts)	PS6-3 (20 pts)	PS6-4 (20 pts)	PS6-5 (20 pts)	Total (100 pts)

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