24-311 NUMERICAL METHODS Fall 03

Carnegie Mellon University

PROBLEM SET 4

Issued: 9/20/03
Due: 9/25/03 10:30am
Weight: 4% of total grade

PS4-1 Formulation of a matrix equation and the usage of Mathcad

An engineer supervises the production of three types of electrical components. Three kind of material—metal, plastic, and rubber—are required for production. The amounts needed to produce each component are shown in the following table.

component	Metal g/component	Plastic g/component	Rubber g/component
1	15	0.25	1.0
2	17	0.33	1.2
3	19	0.42	1.6

If totals of 2.12, 0.0434, and 0.164kg of metal, plastic, and rubber, respectively, are available each day, how many component can be produced per day? Write down a matrix equation and solve it using Mathcad. Name this Mathcad file `ps4-1.mcd`.

In your hand-in directory on AFS, make a new directory called ps4 (in lower case) and copy your Mathcad file. Copy your Mathcad file in the ps4 directory in your hand-in directory on AFS. Also, hand in the printout of the Mathcad file. (Include the title of this problem and your name, e.g., "Metal, plastic and rubber by Kenji Shimada," at the top of your Mathcad file.)

PS4-2 Gauss elimination

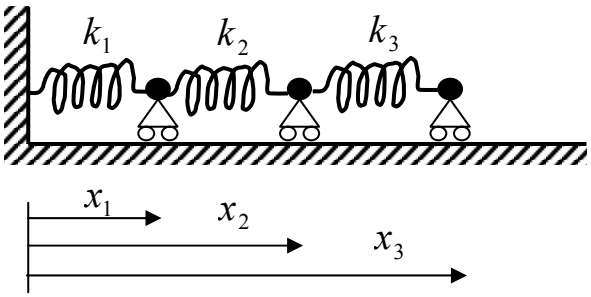
(1) Solve the following matrix equation using Gauss elimination. Show all the intermediate steps.

$$\begin{bmatrix} 4 & 1 & -1 \\ 5 & 1 & 2 \\ 6 & 1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -2 \\ 4 \\ 6 \end{Bmatrix}$$

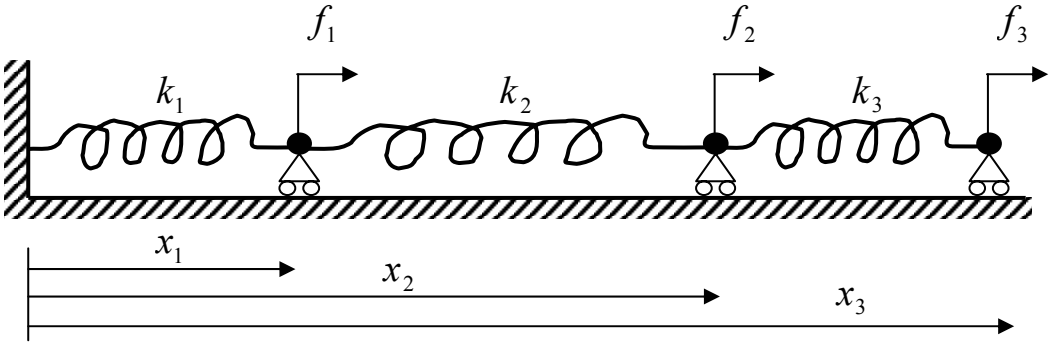
(2) Solve the following matrix equation using Gauss elimination. Find all the solutions, (x_1, x_2, x_3), that satisfy the set of equations. Show all the intermediate steps.

$$\begin{aligned} x_1 - x_2 - 3x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 2 \\ x_1 + 2x_2 + 4x_3 &= 1 \end{aligned}$$

PS4-3 Consider a system of three springs with spring constants $k_1 = 1\ \text{N/m}$, $k_2 = 1\ \text{N/m}$ and $k_3 = 2\ \text{N/m}$ as shown below. The stable positions of the two right-hand-side ends of the springs are $x_1 = 1$, $x_2 = 2$ and $x_3 = 3$. Suppose three external forces $f_1 = 2\ \text{N}$, $f_2 = -1\ \text{N}$ and $f_3 = 2\ \text{N}$ are applied, what are the new positions of the ends of the springs, x_1, x_2 and x_3?



Initial state



After three forces are applied

(1) The condition of force balance gives three linear algebraic equations to be satisfied. What is the matrix equation to be solved in order to find the positions of the ends of the springs, x_1, x_2 and x_3, after three external forces, f_1, f_2, and f_3, are applied?

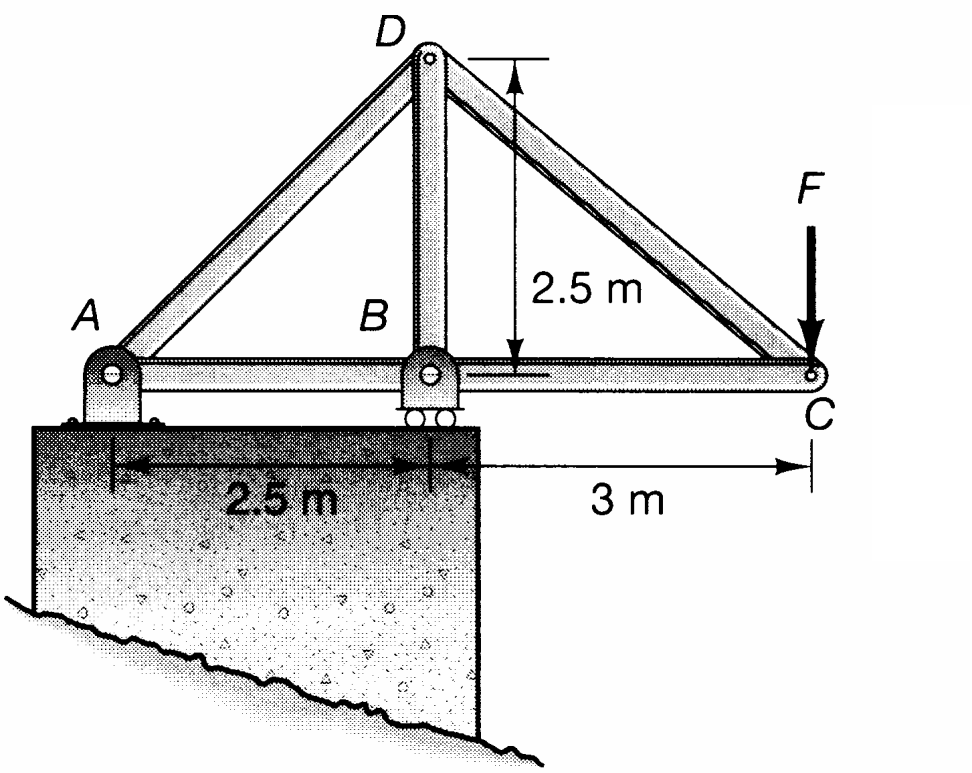
(2) Solve the matrix equation for external forces using Gauss elimination. Show the complete derivation process for full credit.

(3) Solve the same matrix equation for external forces using Gauss elimination with partial pivoting. Show the complete derivation process for full credit.

PS4-4 Formulation of a matrix equation and the usage of Mathcad

Finding the forces associated with a statically determinate truss is an important problem in structural engineering. An overhanging roof-support structure is designed as a simple truss to hold a load of F = 5kN. Computer the reactions at A and B and the force in each member. Also, state whether the members are in tension or in compression. Write down a matrix equation and solve it using Mathcad.

Name this Mathcad file `ps4-4.mcd` and copy the file to the `ps4` directory. Also, hand in the printout of the Mathcad file. (Include the title of this problem and your name, e.g., "Truss Problem by Kenji Shimada," at the top of your Mathcad file.)



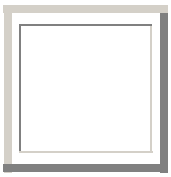
PS4-5 Gauss-Jordan elimination

(1) Solve the following matrix equation using Gauss-Jordan elimination

$$\begin{bmatrix} 2 & 1 & -1 \\ 5 & 2 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -4 \\ 5 \end{Bmatrix}$$

(2) Solve the same matrix equation using Gauss-Jordan elimination with partial pivoting

PS4



The first letter of your LAST name

First Name _____ Last Name _____

PS4-1 (20 pts)	PS4-2 (20 pts)	PS4-3 (20 pts)	PS4-4 (20 pts)	PS4-5 (20 pts)	Total (100 pts)

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