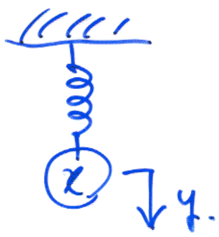
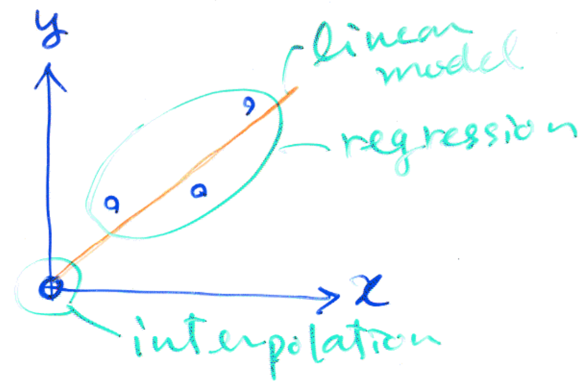
Curve Fitting





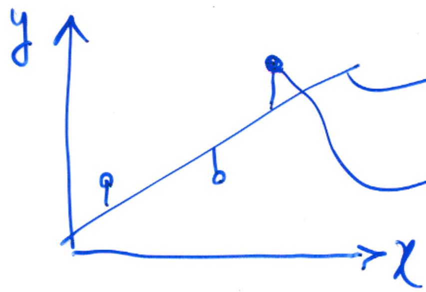
Curve fitting
- regression (least square)
- interpolation (Newton/ Spline/ Lagrange)

Model
- poly
  - linear
  - quadratic
  - cubic
  - ⋮
- non-poly
  - $\sin x$, $\cos x$
  - e^x
  - ⋮

Taylor series.

Dimension
- 1D : curve fitting
- 2D : surface fitting
- ⋮

Least-square Regression



$$y = a_0 + a_1 x$$

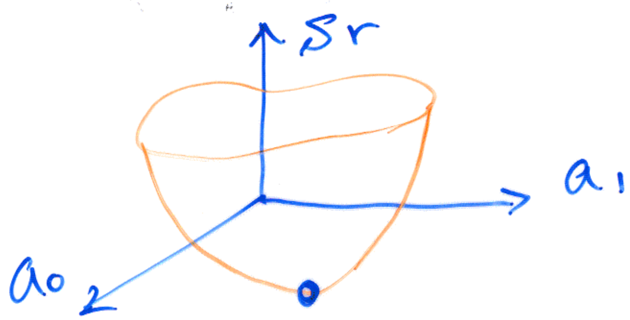
$$y_i = a_0 + a_1 x_i + \underbrace{e_i}_{\text{error}}$$

$$S_r = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

$$\boxed{\text{Find } a_0 \text{ and } a_1 \text{ that minimize } S_r}$$

$$\Downarrow$$

2D optimization problem



minimum

$$\frac{\partial S_r}{\partial a_0} = \frac{\partial S_r}{\partial a_1} = 0$$

$$\Downarrow$$

linear algebraic equation

$$[2 \times 2] \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{Bmatrix} \cdot \\ \cdot \end{Bmatrix}$$

Interpolation

Newton's / Spline / Lagrange's

Reformulation of $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

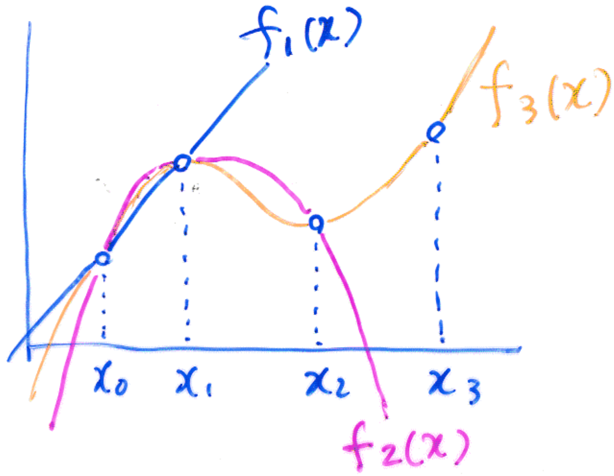
Newton's

cubic

$$f_3(x) = f(x_0) + f[x_1, x_0](x - x_0) \longrightarrow f_1(x)$$

$$+ f[x_2, x_1, x_0](x - x_0)(x - x_1) \longrightarrow f_2(x)$$

$$+ \underbrace{f[x_3, x_2, x_1, x_0]}_{\text{f-d-d approx of derivatives}}(x - x_0)(x - x_1)(x - x_2)$$



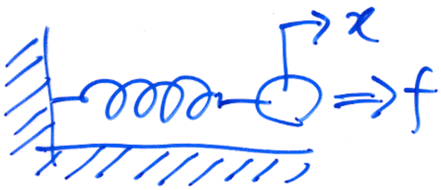
$$\begin{array}{llll}
f(x_0) & & & \\
 & > f[x_1, x_0] & & \\
f(x_1) & & > f[x_2, x_1, x_0] & \\
 & > f[x_2, x_1] & & > f[x_3, x_2, x_1, x_0] \\
f(x_2) & & > f[x_3, x_2, x_1] & \\
 & > f[x_3, x_2] & & \\
f(x_3) & & &
\end{array}$$

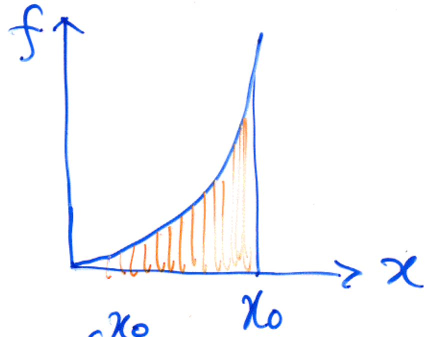
e.g.) $$f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0}$$

* Problems with higher order interpolation

- round-off errors
- overshoot (wiggling)

Numerical Integration





$$E = \frac{1}{2} k x_0^2 \longrightarrow E = \int_0^{x_0} f(x)\,dx$$

Newton-Cotes Integration

(derived from Lagrange's interpolating poly)

1. Zero order (constant)
2. 1st order (trapezoidal)
3. 2nd order (simpson's 1/3)
4. 3rd order (simpson's 3/8)

$$\boxed{I = (x_2 - x_0)\frac{f(x_0) + 4f(x_1) + f(x_2)}{6}}$$

PS10-2

$$I = \int_{x_0}^{x_2} f(x)\,dx \simeq \int_{x_0}^{x_2} f_2(x)\,dx$$

$$= \int_{x_0}^{x_2} \left[L_0 x_0 + L_1 x_1 + L_2 x_2 \right] dx$$

needs $x_1 = \dfrac{x_0 + x_2}{2}$ to obtain

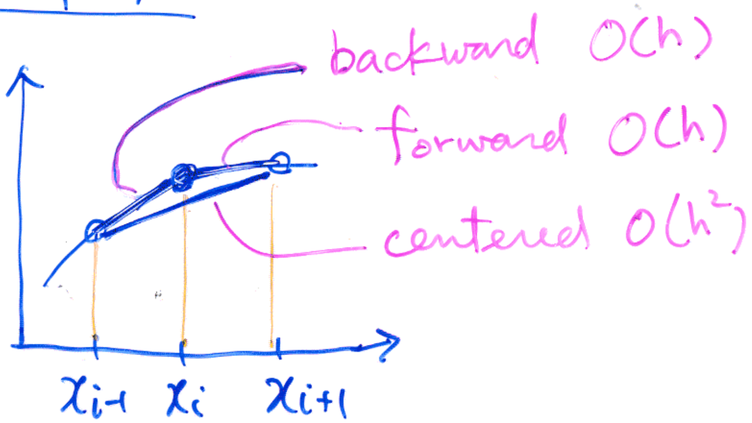
Numerical Differentiation

Taylor Series

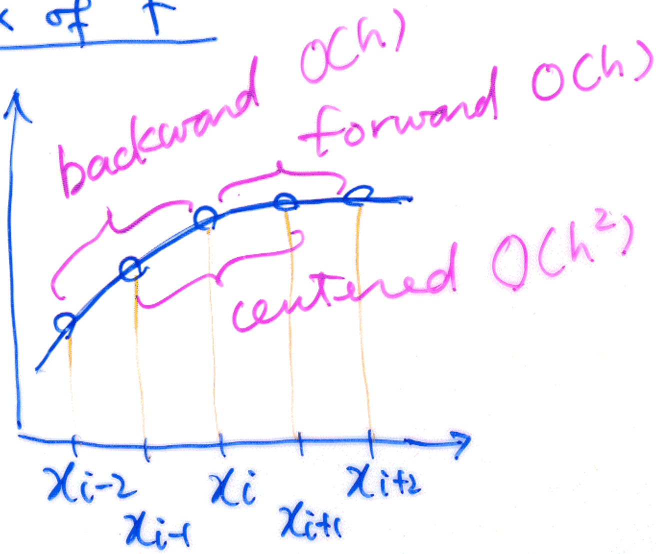
$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \cdots + \frac{f^{(n)}(x_i)}{n!}h^n + \underbrace{\frac{f^{(n+1)}(\xi)}{(n+1)!}h^{n+1}}$$

truncation error $R_n = O(h^{n+1})$

f-d-d approx of f'



f-d-d approx of f''



high accuracy approx of f' and f''

e.g.) f' with 3 points forward

$\longrightarrow O(h^2)$