ME 24-221
Thermodynamics I

Some Useful Formulae

1 Control Mass

Continuity Equation

$$m = \text{constant}$$

First Law

$$U_2 - U_1 + \frac{m\left(\mathbf{V}_2^2 - \mathbf{V}_1^2\right)}{2} + mg\left(Z_2 - Z_1\right) = {}_1Q_2 \;-\; {}_1W_2$$

Compression-expansion work

$$_1W_2 = \int_1^2 PdV$$

For polytropic procees, $PV^n = c$,

$$\begin{aligned} {}_1W_2 &= \frac{P_2V_2 - P_1V_1}{1-n} \quad n \neq 1 \\ &= P_1V_1 \ln\frac{V_2}{V_1} \quad n = 1 \end{aligned}$$

Second Law

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + {}_1S_{2\,gen}$$

For isothermal process

$$\int_1^2 \frac{\delta Q}{T} = \frac{{}_1Q_2}{T}$$

For reversible process

$$_1S_{2\,gen} = 0$$

For adiabatic process

$$_1Q_2 = 0$$

Therefore, for a reversible adiabatic process

$$\begin{aligned} S_2 - S_1 &= 0 \\ s_2 - s_1 &= 0 \end{aligned}$$

Therefore, a reversible adiabatic process is an isentropic process.

2 Control Volume

2.1 Steady State Steady Flow (SSSF)

Continuity

$$\sum_i \dot{m}_i - \sum_e \dot{m}_e = 0$$

First Law

$$\sum_i \dot{m}_i \left(h_i + \frac{\mathbf{V}_i^2}{2} + gZ_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{\mathbf{V}_e^2}{2} + gZ_e\right) + \dot{Q}_{cv} - \dot{W}_{cv} = 0$$

Second Law

$$\sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \sum_j \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen} = 0$$

Reversible process

$$\dot{S}_{gen} = 0$$

Adiabatic process

$$\dot{Q} = 0$$

For one inlet-one outlet device, a reversible adiabatic process is therefore an isentropic process, with

$$s_i = s_e$$

2.2 Uniform State Uniform Flow (USUF)

Continuity

$$(m_2 - m_1) = \sum_i m_i - \sum_e m_e$$

First Law

$$m_2 \left(u_2 + \frac{\mathbf{V}_2^2}{2} + gZ_2\right) - m_1 \left(u_1 + \frac{\mathbf{V}_1^2}{2} + gZ_1\right) = \sum_i m_i \left(h_i + \frac{\mathbf{V}_i^2}{2} + gZ_i\right) - \sum_e m_e \left(h_e + \frac{\mathbf{V}_e^2}{2} + gZ_e\right) + Q_{cv} - W_{cv}$$

Second Law

$$m_2 s_2 - m_1 s_1 = \sum_i m_i s_i - \sum_e m_e s_e + \int_0^t \frac{\dot{Q}_{cv}}{T} dt + {}_1 S_{2\ gen}$$

3 Gibbs Equation

$$\begin{aligned} T ds &= du + P dv \\ &= dh - v dP \end{aligned}$$

This equation holds true for all simple compressible substances.

4 Properties of Pure Substances

4.1 Vapor-Liquid Phase Equilibrium

For a specific property ϕ (such as h,u,v,s etc) under the dome

$$\phi = \phi_f + x\phi_{fg}$$

4.2 Ideal Gas

Ideal Gas Equations of State

$$Pv = RT$$

$$\begin{aligned}
du &= C_v dT \\
u_2 - u_1 &= \int_1^2 C_v dT \\
&= C_v \left(T_2 - T_1\right) \quad \text{if } \mathrm{C_v} \text{ is constant}
\end{aligned}$$

$$\begin{aligned}
dh &= C_p dT \\
h_2 - h_1 &= \int_1^2 C_p dT \\
&= C_p \left(T_2 - T_1\right) \quad \text{if } \mathrm{C_p} \text{ is constant}
\end{aligned}$$

Specific Heats and Ideal Gas Constants

$$\begin{aligned}
C_p - C_v &= R \\
R = \overline{R}/M & \\
\frac{C_p}{C_v} = k &
\end{aligned}$$

Entropy Relationships

$$\begin{aligned}
ds &= C_p \frac{dT}{T} - R\frac{dP}{P} \\
s_2 - s_1 &= \int_1^2 C_p \frac{dT}{T} - R\ln\frac{P_2}{P_1} \\
&= C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \quad \text{if constant } \mathrm{C_p} \\
&= s^0_{T_2} - s^0_{T_1} - R\ln\frac{P_2}{P_1} \quad \text{otherwise}
\end{aligned}$$

$$\begin{aligned}
ds &= C_v \frac{dT}{T} + R\frac{dv}{v} \\
s_2 - s_1 &= \int_1^2 C_v \frac{dT}{T} + R\ln\frac{v_2}{v_1} \\
&= C_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \quad \text{if constant } \mathrm{C_v}
\end{aligned}$$

Isentropic Process for Ideal Gas
For variable specific heats

$$\frac{P_1}{P_2} = \frac{P_{r1}}{P_{r2}}$$
$$\frac{v_1}{v_2} = \frac{v_{r1}}{v_{r2}}$$

For constant specific heats

$$Pv^k = \text{constant}$$
$$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^k$$
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$
$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

The above four relationships also hold for a reversible polytropic process with the polytropic exponent n replacing k.

4.3 Incompressible Substance

Equation of State

$$du = dh = CdT$$
$$u_2 - u_1 = h_2 - h_1 = \int_1^2 CdT$$
$$= C\left(T_2 - T_1\right) \quad \text{if C is constant}$$

Entropy Relationships

$$ds = \frac{du}{T}$$
$$s_2 - s_1 = \int_1^2 C\frac{dT}{T}$$
$$= C\ln\frac{T_2}{T_1} \quad \text{if C is constant}$$

C=constant can usually be assumed for incompressible substances.

5 Heat Engines, Heat Pumps and Refrigerators

Thermal efficiency of heat engine

$$\begin{aligned}\eta_{th} &= \frac{W_{net}}{Q_H} \\ &= \frac{Q_H - Q_L}{Q_H}\end{aligned}$$

Coefficient of Performance (C.O.P) of Heat Pump

$$\begin{aligned}\beta' &= \frac{Q_H}{W_{net}} \\ &= \frac{Q_H}{Q_H - Q_L}\end{aligned}$$

Coefficient of Performance (C.O.P) of Refrigerator

$$\begin{aligned}\beta &= \frac{Q_L}{W_{net}} \\ &= \frac{Q_L}{Q_H - Q_L}\end{aligned}$$

Carnot Cycle

$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$$

$$\begin{aligned}\eta_{th} &= 1 - \frac{T_L}{T_H} \\ \beta' &= \frac{T_H}{T_H - T_L} \\ \beta &= \frac{T_L}{T_H - T_L}\end{aligned}$$

6 Isentropic Efficiency of Engineering Devices

$$\begin{aligned}\eta_{\text{turbine}} &= \frac{w}{w_s} \\ \eta_{\text{compressor}} &= \frac{w_s}{w} \\ \eta_{\text{nozzle}} &= \frac{\mathbf{V}_e^2/2}{\mathbf{V}_{es}^2/2}\end{aligned}$$

7 Irreversibility and Availability

Availability:

$$\Psi = (h - T_0 s) - \left(h_{ref} - T_0 s_{ref}\right)$$

Irreversibility:

$$\dot{I} = T_0 \dot{S}_{gen}$$

Work in an SSSF process with heat gain $\dot{Q}_H$ from T_H and heat loss $\dot{Q}_0$ to T_0:

$$\dot{W}_{net} = \dot{Q}_H \left(1 - \frac{T_0}{T_H}\right) + \dot{m}\left(\Psi_i - \Psi_e\right) - \dot{I}$$

Second Law Efficiency (Effectiveness)

Turbine:

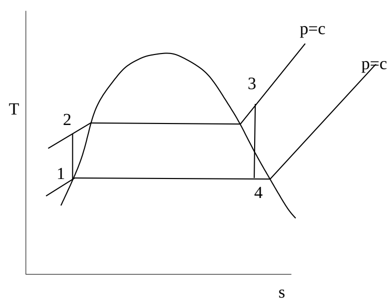
$$\eta_2 = \frac{\dot{W}_{net}}{\dot{m}\left(\Psi_i - \Psi_e\right)}$$

Compressor:

$$\eta_2 = \frac{\dot{m}\left(\Psi_i - \Psi_e\right)}{\dot{W}_{net}}$$

8 Power and Refrigeration Cycles

8.1 Ideal Rankine Cycle



- 1-2:Isentropic compression in pump
- 2-3: Constant pressure heat addition in boiler
- 3-4: Isentropic expansion in turbine
- 4-1: Constant pressure heat rejection in condenser
- Working fluid is usually water or other two-phase substance

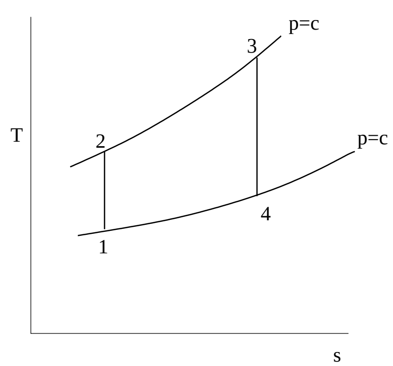
Work and heat transfer formulae:

$$
\begin{aligned}
w_p &= h_1 - h_2 \\
&= v_1 \left(P_1 - P_2\right) \\
q_b &= h_3 - h_2 \\
w_T &= h_3 - h_4 \\
q_c &= h_1 - h_4
\end{aligned}
$$

Also

$$
\begin{aligned}
s_1 &= s_2 \\
s_3 &= s_4
\end{aligned}
$$

8.2 Ideal Air Standard Brayton Cycle



- 1-2:Isentropic compression in compressor
- 2-3: Constant pressure heat addition in heat exchanger
- 3-4: Isentropic expansion in turbine
- 4-1: Constant pressure heat rejection in heat exchanger
- Working fluid idealized to be air

Work and heat transfer formulae:

$$
\begin{aligned}
w_C &= h_1 - h_2 \\
q_h &= h_3 - h_2 \\
w_T &= h_3 - h_4 \\
q_c &= h_1 - h_4
\end{aligned}
$$

Also

$$
\begin{aligned}
s_1 &= s_2 \\
s_3 &= s_4
\end{aligned}
$$

For constant C_p

$$\begin{aligned} w_C &= C_p\left(T_1 - T_2\right) \\ q_h &= C_p\left(T_3 - T_2\right) \\ w_T &= C_p\left(T_3 - T_4\right) \\ q_c &= C_p\left(T_1 - T_4\right) \end{aligned}$$

Also

$$\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \\ \frac{T_4}{T_3} &= \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} \end{aligned}$$

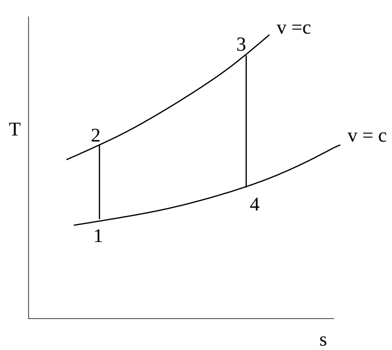
For variable C_p

$$\begin{aligned} \frac{P_{r2}}{P_{r1}} &= \frac{P_2}{P_1} \\ \frac{P_{r4}}{P_{r3}} &= \frac{P_4}{P_3} \end{aligned}$$

Thermal efficiency of Brayton cycle (For constant C_p)

$$\begin{aligned} \eta_{th} &= 1 - \frac{T_1}{T_2} \\ &= 1 - \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} \end{aligned}$$

8.3 Ideal Air Standard Otto Cycle



- Otto cycle is a piston-cylinder cycle (control mass)
- Models spark ignition (SI) engines
- 1-2:Isentropic compression

- 2-3: Constant volume heat addition
- 3-4: Isentropic expansion
- 4-1: Constant volume heat rejection
- Working fluid idealized to be air

Process 1-2:

$$\begin{aligned} s_1 &= s_2 \\ \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad if\ C_p\ constant \\ \frac{T_2}{T_1} &= \left(\frac{V_1}{V_2}\right)^{k-1} \quad if\ C_p\ constant \\ u_2 - u_1 &= -{}_1w_2 \\ &= C_v\left(T_2 - T_1\right) \quad if\ C_p\ constant \end{aligned}$$

Process 2-3:

$$\begin{aligned} u_3 - u_2 &= {}_2q_3 \\ &= C_v\left(T_3 - T_2\right) \quad if\ C_p\ constant \end{aligned}$$

Process 3-4:

$$\begin{aligned} s_3 &= s_4 \\ \frac{T_4}{T_3} &= \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} \quad if\ C_p\ constant \\ \frac{T_4}{T_3} &= \left(\frac{V_3}{V_4}\right)^{k-1} \quad if\ C_p\ constant \\ u_4 - u_3 &= -{}_3w_4 \\ &= C_v\left(T_4 - T_3\right) \quad if\ C_p\ constant \end{aligned}$$

Process 4-1:

$$\begin{aligned} u_1 - u_4 &= {}_1q_4 \\ &= C_v\left(T_1 - T_4\right) \quad if\ C_p\ constant \end{aligned}$$

Thermal efficiency of Otto cycle (For constant C_p)

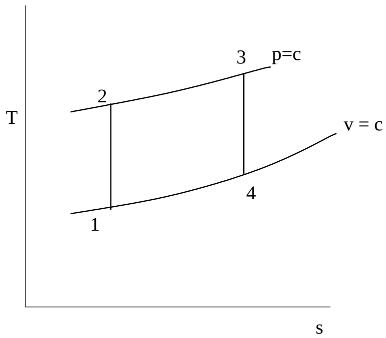
$$\begin{aligned} \eta_{th} &= 1 - \frac{T_1}{T_2} \\ &= 1 - \left(\frac{V_1}{V_2}\right)^{1-k} \\ &= 1 - (r_v)^{1-k} \end{aligned}$$

where r_v is the compression ratio.

Mean Effective Pressure:

$$mep = \frac{w_{net}}{v_1 - v_2}$$

8.4 Ideal Air Standard Diesel Cycle



- Diesel cycle is a piston-cylinder cycle (control mass)
- Models compression ignition (CI) engines
- 1-2:Isentropic compression
- 2-3: Constant pressure heat addition
- 3-4: Isentropic expansion
- 4-1: Constant volume heat rejection
- Working fluid idealized to be air

Process 1-2:

$$\begin{aligned}
s_1 &= s_2 \\
\frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad if\ C_p\ constant \\
\frac{T_2}{T_1} &= \left(\frac{V_1}{V_2}\right)^{k-1} \quad if\ C_p\ constant \\
u_2 - u_1 &= -{}_1w_2 \\
&= C_v\left(T_2 - T_1\right) \quad if\ C_p\ constant
\end{aligned}$$

Process 2-3:

$$\begin{aligned}
h_3 - h_2 &= {}_2q_3 \\
&= C_p\left(T_3 - T_2\right)
\end{aligned}$$

Process 3-4:

$$\begin{aligned}
s_3 &= s_4 \\
\frac{T_4}{T_3} &= \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} \quad if\ C_p\ constant \\
\frac{T_4}{T_3} &= \left(\frac{V_3}{V_4}\right)^{k-1} \quad if\ C_p\ constant \\
u_4 - u_3 &= -{}_3w_4 \\
&= C_v\left(T_4 - T_3\right) \quad if\ C_p\ constant
\end{aligned}$$

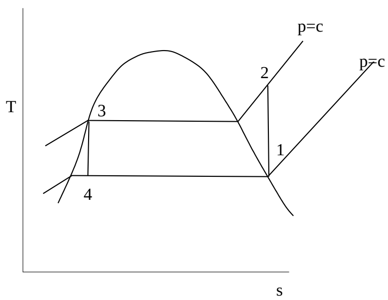
Process 4-1:

$$\begin{aligned} u_1 - u_4 &= {}_1q_4 \\ &= C_v\left(T_1 - T_4\right) \;\; if\; C_p\; constant \end{aligned}$$

Thermal efficiency of Otto cycle (For constant C_p)

$$\eta_{th} = 1 - \frac{C_v\left(T_4 - T_1\right)}{C_p\left(T_3 - T_2\right)}$$

8.5 Ideal Vapor Refrigeration Cycle



- Simply Rankine cycle run backwards
- Note how points 1,2,3,4 have been shifted
- 1-2:Isentropic compression in compressor
- 2-3: Constant pressure heat loss at high-temperature source
- 3-4: Isentropic expansion in expansion valve
- 4-1: Constant pressure heat gain in evaporator
- Working fluid is usually refrigerant or other two-phase substance