ME 24-221
THERMODYNAMICS I

Solutions to extra problems in Chapter 9:
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J. Murthy

9.10 Atmospheric air at -45°C, 60 kPa enters the front diffuser of a jet engine with a velocity of 900 km/h and frontal area of 1 m^2. After the adiabatic diffuser the velocity is 20 m/s. Find the diffuser exit temperature and the maximum pressure possible.

C.V. Diffuser, SSSF single inlet and exit flow, no work or heat transfer.

Energy Eq.: $h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2$, and $h_e - h_i = C_p(T_e - T_i)$

Entropy Eq.: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$ (Reversible, adiabatic)

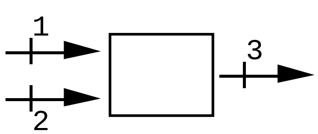
Heat capacity and ratio of specific heats from Table A.5 in the energy equation then gives:

$$1.004[\,T_e - (-45)] = 0.5[(900\times1000/3600)^2 - 20^2\,]/1000 = 31.05 \text{ kJ/kg}$$

$$\Rightarrow\quad T_e = -14.05\ °C = \mathbf{259.1\ K}$$

Constant s: $P_e = P_i\,(T_e/T_i)^{\frac{k}{k-1}} = 60\ (259.1/228.1)^{3.5} = \mathbf{93.6\ kPa}$

9.12 Two flowstreams of water, one at 0.6 MPa, saturated vapor, and the other at 0.6 MPa, 600°C, mix adiabatically in a SSSF process to produce a single flow out at 0.6 MPa, 400°C. Find the total entropy generation for this process.



1: B.1.2 $h_1 = 2756.8$ kJ/kg, $s_1 = 6.760$ kJ/kg K
2: B.1.3 $h_2 = 3700.9$ kJ/kg, $s_2 = 8.2674$ kJ/kg K
3: B.1.3 $h_3 = 3270.3$ kJ/kg, $s_3 = 7.7078$ kJ/kg K

Cont.: $\dot m_3 = \dot m_1 + \dot m_2$, Energy Eq.: $\dot m_3 h_3 = \dot m_1 h_1 + \dot m_2 h_2$

$\Rightarrow\ \dot m_1/\dot m_3 = (h_3 - h_2) / (h_1 - h_2) = 0.456$

Entropy Eq.: $\dot m_3 s_3 = \dot m_1 s_1 + \dot m_2 s_2 + \dot S_{gen}$ $\Rightarrow$

$$\dot S_{gen}/\dot m_3 = s_3 - (\dot m_1/\dot m_3)\, s_1 - (\dot m_2/\dot m_3)\, s_2$$

$$= 7.7078 - 0.456\times6.760 - 0.544\times8.2674 = \mathbf{0.128\ kJ/kg\ K}$$

9.19 Air at 327°C, 400 kPa with a volume flow 1 m^3/s runs through an adiabatic turbine with exhaust pressure of 100 kPa. Neglect kinetic energies and use constant specific heats. Find the lowest and highest possible exit temperature. For each case find also the rate of work and the rate of entropy generation.

C.V Turbine. SSSF, single inlet and exit flows, q = 0.

Inlet state: (T, P) $\quad v_i = RT_i / P_i = 0.287 \times 600/400 = 0.4305\ m^3/kg$

$\dot{m} = \dot{V}/v_i = 1/0.4305 = 2.323$ kg/s

The lowest exit T is for max work out i.e. reversible case

Constant s => $\quad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 600 \times (100/400)^{0.2857} = \mathbf{403.8\ K}$

$\Rightarrow \quad w = h_i - h_e = C_{Po}(T_i - T_e) = 1.004 \times (600 - 403.8) = 197$ kJ/kg

$\dot{W}_T = \dot{m}w = 0.4305 \times 197 = \mathbf{457.6\ kW}$ $\quad$ and $\quad$ $\dot{\mathbf{S}}_{\mathbf{gen}} \mathbf{= 0}$

Highest exit T occurs when there is no work out, throttling

$q = \varnothing; \quad w = \varnothing \quad \Rightarrow \quad h_i - h_e = 0 \quad \Rightarrow \quad T_e = T_i = \mathbf{600\ K}$

$\dot{S}_{gen} = \dot{m}(s_e - s_i) = -\dot{m}R \ln P_e/P_i = -2.323 \times 0.287 \ln \frac{100}{400} = \mathbf{0.924\ kW/K}$

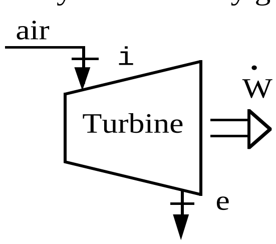
9.21 Air enters a turbine at 800 kPa, 1200 K, and expands in a reversible adiabatic process to 100 kPa. Calculate the exit temperature and the work output per kilogram of air, using

a. The ideal gas tables, Table A.7

b. Constant specific heat, value at 300 K from table A.5

c. Constant specific heat, value at an intermediate temperature from Fig. 5.1

Discuss why the method of part (b) gives a poor value for the exit temperature and yet a relatively good value for the work output.



C.V. Air turbine. Adiabatic; q = 0, reversible: $s_{gen} = 0$

Energy: $\quad w_T = h_i - h_e$, $\quad$ Entropy Eq.: $\quad s_e = s_i$

a) Table A.7: $\quad h_i = 1277.8, \quad P_{ri} = 191.174$

$\Rightarrow P_{re} = P_{ri} \times P_e/P_i = 191.174 \times 100/800 = 23.897$

$\Rightarrow T_e = \mathbf{706\ K}, \quad h_e = 719.9$ kJ/kg

$w = h_i - h_e = \mathbf{557.9\ kJ/kg}$

b) $\quad T_e = T_i (P_e/P_i)^{\frac{k-1}{k}} = 1200 \left(\frac{100}{800}\right)^{0.286} = \mathbf{662.1\ K}$

$w = C_{Po}(T_i - T_e) = 1.004(1200 - 662.1) = \mathbf{539.8\ kJ/kg}$

c) Fig. 5.10 at ~ 1000 K: $\bar{C}_{Po} \sim 32.5$, $\bar{C}_{Vo} = \bar{C}_{Po} - \bar{R} \sim 24.2$

$k = \bar{C}_{Po}/\bar{C}_{Vo} \sim 1.343, \quad T_e = 1200\ (100/800)^{0.255} = \mathbf{706.1\ K}$

$w = (32.5/28.97)(1200 - 706.1) = \mathbf{554.1\ kJ/kg}$

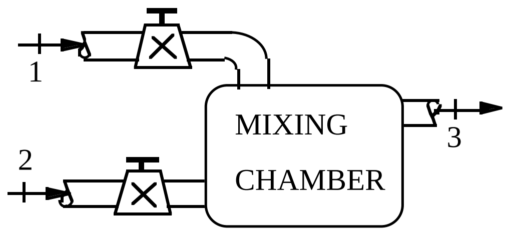
In b) k = 1.4 is too large and C_p too small.

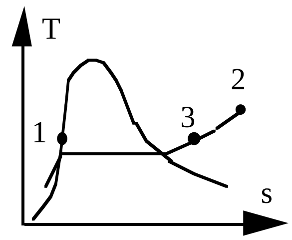
9.23 A supply of 5 kg/s ammonia at 500 kPa, 20°C is needed. Two sources are available one is saturated liquid at 20°C and the other is at 500 kPa, 140°C. Flows from the two sources are fed through valves to an insulated SSSFmixing chamber, which then produces the desired output state. Find the two source mass flow rates and the total rate of entropy generation by this setup.

C.V. mixing chamber + valve. SSSF, no heat transfer, no work.

Continuity Eq.: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$; Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$





State 1: Table B.2.1 $h_1 = 273.4$ kJ/kg, $s_1 = 1.0408$ kJ/kg K

State 2: Table B.2.2 $h_2 = 1773.8$ kJ/kg, $s_2 = 6.2422$ kJ/kg K

State 3: Table B.2.2 $h_3 = 1488.3$ kJ/kg, $s_3 = 5.4244$ kJ/kg K

$$\dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_2) h_2 = \dot{m}_3 h_3 \quad => \quad \dot{m}_1 = \dot{m}_3 \frac{h_3 - h_2}{h_1 - h_2} = 0.952 \text{ kg/s}$$

$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 4.05$ kg/s

$\dot{S}_{gen} = 5 \times 5.4244 - 0.95 \times 1.0408 - 4.05 \times 6.2422 =$ **0.852 kW/K**

9.49 A certain industrial process requires a steady 0.5 kg/s of air at 200 m/s, at the condition of 150 kPa, 300 K. This air is to be the exhaust from a specially designed turbine whose inlet pressure is 400 kPa. The turbine process may be assumed to be reversible and polytropic, with polytropic exponent n = 1.20.

a) What is the turbine inlet temperature?

b) What are the power output and heat transfer rate for the turbine?

c) Calculate the rate of net entropy increase, if the heat transfer comes from a source at a temperature 100°C higher than the turbine inlet temperature.

C.V. Turbine, this has heat transfer, PV^n = Const., n = 1.2

Air table A.5: C_p = 1.004 kJ/kg-K, R = 0.287 kJ/kg-K

Exit: T_e = 300K, P_e = 150 kPa, $\mathbf{V}_e$ = 200 m/s

a) Process polytropic: $T_e / T_i = (P_e/P_i)^{\frac{n-1}{n}} \Rightarrow T_i = 353.3 \text{ K}$

b) 1st Law SSSF: $\dot{m}_i(h+\mathbf{V}^2/2)_{in} + \dot{Q} = \dot{m}_{ex}(h+\mathbf{V}^2/2)_{ex} + \dot{W}_T$

Reversible shaft work in a polytropic process, Eq.9.15 and Eq.9.20

$$w_T = -\int v\,dP + (\mathbf{V}_i^2 - \mathbf{V}_e^2)/2 = -\frac{n}{n-1}(P_e v_e - P_i v_i) + (\mathbf{V}_i^2 - \mathbf{V}_e^2)/2$$

$$= -\frac{n}{n-1}R(T_e - T_i) - \mathbf{V}_e^2/2 = 71.8 \text{ kJ/kg}$$

$\dot{W}_T = \dot{m}w_T =$ **35.9 kW**

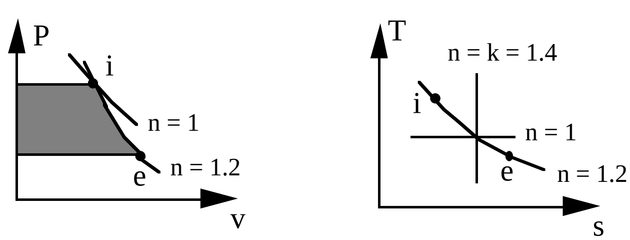
Assume Constant Specific Heat

$\dot{Q} = \dot{m}[C_P(T_e - T_i) - \mathbf{V}_e^2/2] + \dot{W}_T =$ **19.2 kW**

c) 2nd Law: $dS_{net}/dt = \dot{m}(s_e - s_i) - \dot{Q}_H/T_H$, $T_H = T_i + 100 = 453.3 \text{ K}$

$$s_e - s_i = C_p \ln\frac{T_e}{T_i} - R\ln\frac{P_e}{P_i} = 0.1174 \text{ kJ/kg-K}$$

$dS_{net}/dt = 0.5\times0.1174 - 19.2/453.3 = 0.0163$ kW/K



9.60 A nozzle is required to produce a flow of air at 200 m/s at 20°C, 100 kPa. It is estimated that the nozzle has an isentropic efficiency of 92%. What nozzle inlet pressure and temperature is required assuming the inlet kinetic energy is negligible?

C.V. Air nozzle: P_e, T_e(real), V_e(real), η_s(real)

For the real process: $h_i = h_e + \mathbf{V}_e^2/2$ or

$T_i = T_e + \mathbf{V}_e^2/2C_{P0} = 293.2 + 200^2/2 \times 1000 \times 1.004 = \mathbf{313.1\ K}$

For the ideal process:

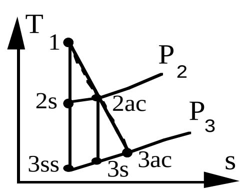
$\mathbf{V}_{es}^2/2 = \mathbf{V}_e^2/2\eta_s = 200^2/2 \times 1000 \times 0.92 = 21.74$ kJ/kg

and $h_i = h_{es} + (\mathbf{V}_{es}^2/2)$

$T_{es} = T_i - \mathbf{V}_{es}^2/(2C_{P0}) = 313.1 - 21.74/1.004 = 291.4$ K

$$\Rightarrow \quad P_i = P_e\,(T_i/T_{es})^{\frac{k}{k-1}} = 100\left(\frac{313.1}{291.4}\right)^{3.50} = \mathbf{128.6\ kPa}$$

9.71 A two-stage turbine receives air at 1160 K, 5.0 MPa. The first stage exit at 1 MPa then enters stage 2, which has an exit pressure of 200 kPa. Each stage has an isentropic efficiency of 85%. Find the specific work in each stage, the overall isentropic efficiency, and the total entropy generation.



C.V. around each turbine for first the ideal and then the actual produces for stage 1:

Ideal T1: $s_{2s}=s_1$ => $P_{r2} = P_{r1}P_2/P_1 = 33.297$

$\Rightarrow$ $h_{2s} = 789.93$; $w_{t1s} = h_{2s} - h_1 = \mathbf{441.04}$

$w_{T1ac} = \eta w_{T1s} = 374.88$ kJ/kg $\Rightarrow$ $h_{2ac} = 856.09$, $P_{r2ac} = 44.57$

$P_{r3} = P_{r2ac}P_3/P_2 = 8.915$ $\Rightarrow$ $h_{3s} = 544.49$, $w_{T2s} = \mathbf{311.6}$

$w_{T2ac} = \eta w_{T2s} = 264.86$ $\Rightarrow$ $h_{3ac} = 591.23$, $s^\circ_{T3ac} = 7.5491$

$T_{2s} = 770$, $T_{2ac} = 830$, $T_{3s} = 540$, $T_{3ac} \cong 585$

For the overall isentropic efficiency we need the isentropic work:

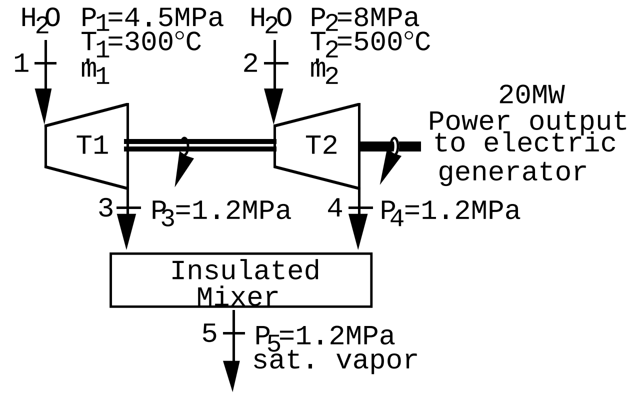
$P_{r3ss} = P_{r1}P_3/P_1 = 6.659$ $\Rightarrow$ $h_{3ss} = 500.97$ $w_{ss} = 730.0$

$\eta = (w_{T1ac} + w_{T2ac})/w_{ss} = \mathbf{0.876}$

$s_{T\,gen} = s_3 - s_1 = s^\circ_{T3} - s^\circ_{T1} - R \ln (P_3/P_1)$

$$= 7.5491 - 8.30626 - 0.287 \ln\left(\frac{200}{5000}\right) = \mathbf{0.1666\ kJ/kg\ K}$$

9.72 A paper mill, shown in Fig. P9.72, has two steam generators, one at 4.5 MPa, 300°C and one at 8 MPa, 500°C. Each generator feeds a turbine, both of which have an exhaust pressure of 1.2 MPa and isentropic efficiency of 87%, such that their combined power output is 20 MW. The two exhaust flows are mixed adiabatically to produce saturated vapor at 1.2 MPa. Find the two mass flow rates and the entropy produced in each turbine and in the mixing chamber.



$h_1 = 2943.1$

$s_1 = 6.2828$

$h_2 = 3398.3$

$s_2 = 6.7240$

$h_5 = 2784.8$

$s_5 = 6.5233$

CV T1:

$w_{T1,s} = h_1 - h_{3s}$

$s_{3s} = s_1$

3s: $x_{3s} = (6.2828 - 2.2165)/4.3067 = 0.9442 \quad \Rightarrow$

$h_{3s} = 798.6 + 0.9442 \times 1986.2 = 2673.9 \quad \Rightarrow \quad w_{T1,s} = 269.2$ kJ/kg

$w_{T1,AC} = 0.87 \times 269.2 = 234.2$,

$h_{3,AC} = 2708.9 = 798.6 + x_{3AC} \times 1986.2 \quad => \; x_{3AC} = 0.9618$

$s_{3AC} = 2.2165 + 0.9618 \times 4.3067 = 6.3586$,

$s_{genT1} = 6.3586 - 6.2828 = 0.0758$ kJ/kg K

CV T2: $w_{T2,s} = h_2 - h_{4s}$, $s_{4s} = s_2 = 6.7240$

4s: $T_{4s} = 226.7°C$, $h_{4s} = 2881.1$, $w_{T2,s} = 517.2$, $w_{T2,AC} = 450.0$

$h_{4,AC} = 2948.3$, $s_{4AC} = 6.8546$,

$s_{genT2} = 6.8546 - 6.7240 = 0.1306$ kJ/kg K

C.V mixer: $\dot{m}_{T1}h_{3AC} + \dot{m}_{T2}h_{4,AC} = (\dot{m}_{T1} + \dot{m}_{T2})h_5$

$\Rightarrow \; (\dot{m}_{T1}/\dot{m}_{Tot})(h_{3,AC} - h_{4,AC}) = h_5 - h_{4,AC}$

$(\dot{m}_{T1}/\dot{m}_{Tot}) = 0.683$, $\quad (\dot{m}_{T2}/\dot{m}_{tot}) = 0.317$

C.V. Total: $\dot{m}_1h_1 + \dot{m}_2h_2 = 20\text{ MW} + \dot{m}_{tot}h_5$

$\dot{m}_{tot} \times 302.598 = 20\text{MW} \quad \Rightarrow \quad \dot{m}_{tot} = 66.094$ kg/s

$\dot{m}_{tot}s_{gen} = \dot{m}_{tot}s_5 - \dot{m}_{T1}s_3 - \dot{m}_{T2}s_4 = 0.00747\ \dot{m}_{tot} =$ **0.494 kW/K**