ME 24-221
THERMODYNAMICS I

Solutions to extra problems in chapter 8
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8.5 Water is used as the working fluid in a Carnot cycle heat engine, where it changes from saturated liquid to saturated vapor at 200°C as heat is added. Heat is rejected in a constant pressure process (also constant T) at 20 kPa. The heat engine powers a Carnot cycle refrigerator that operates between -15°C and +20°C. Find the heat added to the water per kg water. How much heat should be added to the water in the heat engine so the refrigerator can remove 1 kJ from the cold space?

Solution:

Carnot cycle:

$q_H = T_H (s_2 - s_1) = h_{fg} = 473.15\ (4.1014) =$ **1940 kJ/kg**

$T_L = T_{sat}\ (20\ \text{kPa}) = 60.06\ ^oC$

$\beta_{ref} = Q_L / W = T_L / (T_H - T_L) = (273 - 15) / (20 - (-15))$

$= 258 / 35 = 7.37$

$W = Q_L / \beta = 1 / 7.37 = 0.136$ kJ

$W = \eta_{HE}\ Q_{H\ H2O}$ $\qquad \eta_{HE} = 1 - 333/473 = 0.29$

$Q_{H\ H2O} = 0.136 / 0.296 =$ **0.46 kJ**

8.12 A cylinder fitted with a piston contains ammonia at 50°C, 20% quality with a volume of 1 L. The ammonia expands slowly, and during this process heat is transferred to maintain a constant temperature. The process continues until all the liquid is gone. Determine the work and heat transfer for this process.

C.V. Ammonia in the cylinder.

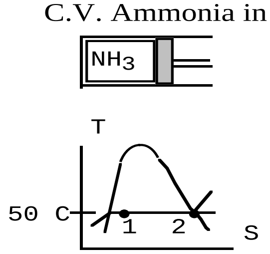


Table B.2.1: $T_1 = 50°C$, $x_1 = 0.20$, $V_1 = 1$ L

$v_1 = 0.001777 + 0.2 \times 0.06159 = 0.014095$

$s_1 = 1.5121 + 0.2 \times 3.2493 = 2.1620$

$m = V_1/v_1 = 0.001/0.014095 = 0.071$ kg

$v_2 = v_G = 0.06336$, $\quad s_2 = s_G = 4.7613$

Process: T = constant to $x_2 = 1.0$, $\quad P$ = constant = 2.033 MPa

${}_1W_2 = \int PdV = Pm(v_2 - v_1) = 2033 \times 0.071 \times (0.06336 - 0.014095) =$ **7.11 kJ**

${}_1Q_2 = \int TdS = Tm(s_2 - s_1) = 323.2 \times 0.071(4.7613 - 2.1620) =$ **59.65 kJ**

or $\quad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$

$h_1 = 421.48 + 0.2 \times 1050.01 = 631.48$, $\quad h_2 = 1471.49$

${}_1Q_2 = 0.071(1471.49 - 631.48) =$ **59.65 kJ**

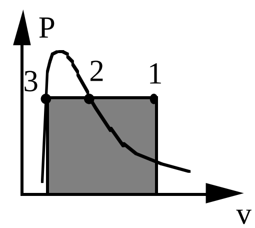
8.14 A cylinder fitted with a frictionless piston contains water. A constant hydraulic pressure on the back face of the piston maintains a cylinder pressure of 10 MPa. Initially, the water is at 700°C, and the volume is 100 L. The water is now cooled and condensed to saturated liquid. The heat released during this process is the Q supply to a cyclic heat engine that in turn rejects heat to the ambient at 30°C. If the overall process is reversible, what is the net work output of the heat engine?

C.V.: H_2O, 1‡3, this is a control mass:

Continuity Eq.: $m_1 = m_3 = m$

Energy Eq.: $m(u_3-u_1) = {}_1Q_3 - {}_1W_3;$

Process: $P = C \Rightarrow {}_1W_3 = \int P\,dV = Pm(v_3-v_1)$



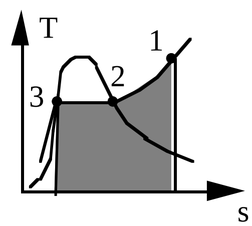
State 1: 700^oC, 10 MPa, $V_1 = 100$ L Table B.1.4

$v_1 = 0.04358\ m^3/kg \Rightarrow m = m_1 = V_1/v_1 = 2.295$ kg

$h_1 = 3870.5$ kJ/kg, $s_1 = 7.1687$ kJ/kg K

State 3: $P_3 = P_1 = 10$ MPa, $x_3 = 0$ Table B.1.2

$h_3 = h_f = 1407.5$ kJ/Kg, $s_3 = s_f = 3.3595$ kJ/Kg K



$${}_1Q_3 = m(u_3-u_1) + Pm(v_3 - v_1) = m(h_3 - h_1)$$

$$= -5652.6 \text{ kJ}$$

Heat transfer to the heat engine:

$Q_H = -{}_1Q_3 = 5652.6$ kJ

Take control volume as total water and heat engine.

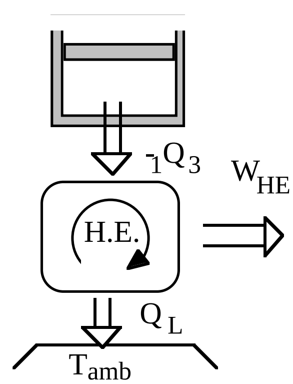
Process: Rev., $\Delta S_{net} = 0$; $T_L = 30^oC$

2^{nd} Law: $\Delta S_{net} = m(s_3 - s_1) - Q_{cv}/T_L$;

$$Q_{cv} = T_o\, m(s_3 - s_1) = -2650.6 \text{ kJ}$$

$\Rightarrow Q_L = -Q_{cv} = 2650.6$ kJ

$W_{net} = W_{HE} = Q_H - Q_L =$ **3002 kJ**



8.22 A heavily-insulated cylinder fitted with a frictionless piston contains ammonia 6°C, 90% quality, at which point the volume is 200 L. The external force on th piston is now increased slowly, compressing the ammonia until its temperature reaches 50°C. How much work is done on the ammonia during this process?

Solution:

C.V. ammonia in cylinder, insulated so assume adiabatic $Q = 0$.

Cont.Eq.: $m_2 = m_1 = m$; Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

State 1: $T_1 = 6^oC$, $x_1 = 0.9$, $V_1 = 200\ L = 0.2\ m^3$

Table B.2.1 saturated vapor, $P_1 = P_g = 534$ kPa

$v_1 = v_f + x_1 v_{fg} = 0.21166\ m^3/kg$,

$u_1 = u_f + x_1 u_{fg} = 207.414 + 0.9 \times 1115.3 = 1211.2$ kJ/kg

$s_1 = s_f + x_1 s_{fg} = 0.81166 + 0.9 \times 4.4425 = 4.810$ kJ/kg-K,

$m_1 = V_1/v_1 = 0.2 / 0.21166 = 0.945$ kg

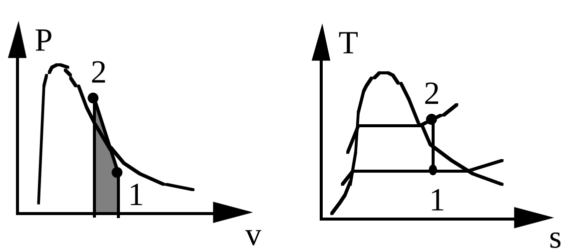
Process: 1‡2 Adiabatic ${}_1Q_2 = 0$ & Reversible ${}_1S_{2\ gen} = 0$ => $s_1 = s_2$

State 2: $T_2 = 50^oC$, $s_2 = s_1 = 4.810$ kJ/kg-K

superheated vapor, interpolate in Table B.2.2 => $P_2 = 1919$ kPa,

$v_2 = 0.0684\ m^3/kg$, $h_2 = 1479.5$ kJ/kg

$u_2 = h_2 - P_2 v_2 = 1479.5 - 1919 \times 0.0684 = 1348.2$ kJ/kg



Energy equation gives the work as

${}_1W_2 = m(u_1 - u_2) = 0.945\ (1211.2 - 1348.2) =$ **-129.4 kJ**

8.27 An insulated cylinder/piston contains R-134a at 1 MPa, 50°C, with a volume of 100 L. The R-134a expands, moving the piston until the pressure in the cylinder has dropped to 100 kPa. It is claimed that the R-134a does 190 kJ of work against the piston during the process. Is that possible?

C.V. R-134a in cylinder. Insulated so assume Q = 0.

State 1: Table B.5.2, $v_1 = 0.02185$, $u_1 = 431.24 - 1000\times0.02185 = 409.4$,

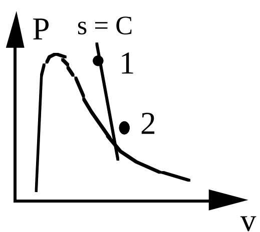
$s_1 = 1.7494$, $m = V_1/v_1 = 0.1/0.02185 = 4.577$ kg

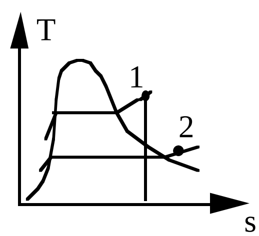
Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = \emptyset - 190 \;\Rightarrow\; u_2 = 367.89$ kJ/kg

State 2: P_2 , u_2 $\Rightarrow$ Table B.5.2: $T_2 = -19.25°C$; $s_2 = 1.7689$ kJ/kg K

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2,gen} = {}_1S_{2,gen} = 0.0893$ kJ/K

This is possible since ${}_1S_{2,gen} > \emptyset$

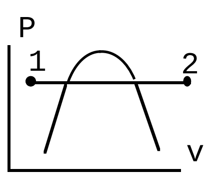


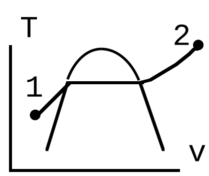


8.29 A mass and atmosphere loaded piston/cylinder contains 2 kg of water at 5 MPa, 100°C. Heat is added from a reservoir at 700°C to the water until it reaches 700°C. Find the work, heat transfer, and total entropy production for the system and surroundings.

C.V. Water. Process: P = const. so ${}_1W_2 = P(V_2 - V_1)$

$U_2 - U_1 = {}_1Q_2 - {}_1W_2$ or ${}_1Q_2 = H_2 - H_1 = m(h_2 - h_1)$





B.1.4: $h_1 = 422.72$, $u_1 = 417.52$

$s_1 = 1.303$, $v_1 = 0.00104$

B.1.3: $h_2 = 3900.1$, $u_2 = 3457.6$

$s_2 = 7.5122$, $v_2 = 0.08849$

${}_1Q_2 = 2(3900.1 - 422.72) =$ **6954.76 kJ**

${}_1W_2 = {}_1Q_2 - m(u_2 - u_1) =$ **874.6 kJ**

$m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_{res} + {}_1S_{2\,gen}$

${}_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2/T_{res} = 2(7.5122 - 1.303) - 6954/973 =$ **5.27 kJ/K**

8.41 A large slab of concrete, 5 × 8 × 0.3 m, is used as a thermal storage mass in a solar-heated house. If the slab cools overnight from 23°C to 18°C in an 18°C house, what is the net entropy change associated with this process?

C.V.: Control mass concrete. $V = 5 \times 8 \times 0.3 = 12\ m^3$

$m = \rho V = 2300 \times 12 = 27600$ kg

${}_1Q_2 = mC\Delta T = 27600 \times 0.65(-5) = -89700$ kJ

$$\Delta S_{SYST} = mC \ln\frac{T_2}{T_1} = 27600 \times 0.65 \ln\frac{291.2}{296.2} = -305.4 \text{ kJ/K}$$

$\Delta S_{SURR} = -{}_1Q_2/T_0 = +89700/291.2 = +308.0$ kJ/K

$\Delta S_{NET} = -305.4 + 308.0 =$ **+2.6 kJ/K**

8.44 A hollow steel sphere with a 0.5-m inside diameter and a 2-mm thick wall contains water at 2 MPa, 250°C. The system (steel plus water) cools to the ambient temperature, 30°C. Calculate the net entropy change of the system and surroundings for this process.

C.V.: Steel + water. This is a control mass.

$m_{STEEL} = (\rho V)_{STEEL} = 8050 \times (\pi/6)\left[(0.504)^3 - (0.5)^3\right] = 12.746$ kg

$\Delta U_{STEEL} = (mC)_{STEEL}(T_2 - T_1) = 12.746 \times 0.48(30 - 250) = -1346$ kJ

$V_{H2O} = (\pi/6)(0.5)^3$, $m = V/v = 6.545 \times 10^{-2}/0.11144 = 0.587$ kg

$v_2 = v_1 = 0.11144 = 0.001004 + x_2 \times 32.889 \Rightarrow x_2 = 3.358 \times 10^{-3}$

$u_2 = 125.78 + 3.358 \times 10^{-3} \times 2290.8 = 133.5$

$s_2 = 0.4639 + 3.358 \times 10^{-3} \times 8.0164 = 0.4638$

$\Delta U_{H_2O} = m_{H_2O}(u_2 - u_1)_{H_2O} = 0.587(133.5 - 2679.6) = -1494.6$

${}_1Q_2 = -1346 + (-1494.6) = -2840.6$

$\Delta S_{TOT} = \Delta S_{STEEL} + \Delta S_{H_2O} = 12.746 \times 0.48 \ln(303.15 / 523.15)$

$+ 0.587(0.4638 - 6.545) =$ **-6.908 kJ/K**

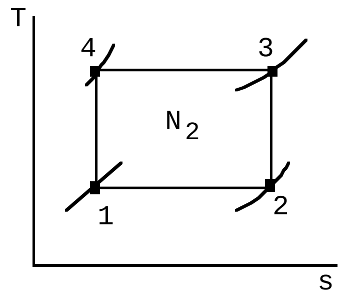
$\Delta S_{SURR} = -{}_1Q_2/T_0 = +2840.6/303.2 =$ **+9.370 kJ/K**

$\Delta S_{NET} = -6.908 + 9.370 =$ **+2.462 kJ/K**

8.47 Consider a Carnot-cycle heat pump having 1 kg of nitrogen gas in a cylinder/piston arrangement. This heat pump operates between reservoirs at 300 K and 400 K. At the beginning of the low-temperature heat addition, the pressure is 1 MPa. During this processs the volume triples. Analyze each of the four processes in the cycle and determine

a. The pressure, volume, and temperature at each point

b. The work and heat transfer for each process



$T_1 = T_2 = \mathbf{300\ K}$, $T_3 = T_4 = \mathbf{400\ K}$,

$P_1 = \mathbf{1\ MPa}$, $V_2 = 3 \times V_1$

a) $P_2V_2 = P_1V_1 \;\Rightarrow\; P_2 = P_1/3 = \mathbf{0.3333\ MPa}$

$$V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.2968 \times 300}{1000} = \mathbf{0.08904\ m^3}$$

$V_2 = \mathbf{0.26712\ m^3}$

$$P_3 = P_2(T_3/T_2)^{\frac{k}{k-1}} = 0.3333\left(\frac{400}{300}\right)^{3.5} = \mathbf{0.9123\ MPa}$$

$$V_3 = V_2 \times \frac{P_2}{P_3} \times \frac{T_3}{T_2} = 0.26712 \times \frac{0.3333}{0.9123} \times \frac{400}{300} = \mathbf{0.1302\ m^3}$$

$$P_4 = P_1(T_3/T_1)^{\frac{k}{k-1}} = 1\left(\frac{400}{300}\right)^{3.5} = \mathbf{2.73707\ MPa}$$

$$V_4 = V_1 \times \frac{P_1}{P_4} \times \frac{T_4}{T_1} = 0.08904 \times \frac{1}{2.737} \times \frac{400}{300} = \mathbf{0.04337\ m^3}$$

b) ${}_1W_2 = {}_1Q_2 = mRT_1 \ln(P_1/P_2)$

$= 1 \times 0.2968 \times 300 \ln(1/0.333) = \mathbf{97.82\ kJ}$

${}_3W_4 = {}_3Q_4 = mRT_3 \ln(P_3/P_4)$

$= 1 \times 0.2968 \times 400 \ln(0.9123/2.737) = \mathbf{-130.43\ kJ}$

${}_2W_3 = -mC_{V0}(T_3 - T_2) = -1 \times 0.7448(400 - 300) = \mathbf{-74.48\ kJ}$

${}_4W_1 = -mC_{V0}(T_1 - T_4) = -1 \times 0.7448(300 - 400) = \mathbf{+74.48\ kJ}$

${}_2Q_3 = \mathbf{0}$, ${}_4Q_1 = \mathbf{0}$

8.52 A rigid storage tank of 1.5 m^3 contains 1 kg argon at 30°C. Heat is then transferred to the argon from a furnace operating at 1300°C until the specific entropy of the argon has increased by 0.343 kJ/kg K. Find the total heat transfer and the entropy generated in the process.

Solution:

C.V. Argon. Control mass. R = 0.20813, m = 1 kg

Energy Eq.: $m (u_2 - u_1) = m C_v (T_2 - T_1) = {}_1Q_2$

Process: V = constant => $v_2 = v_1$

State 1: $P_1 = mRT/V = 42.063$ kPa

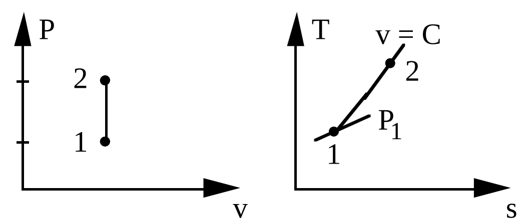
State 2: $s_2 = s_1 + 0.343$,

$$s_2 - s_1 = C_p \ln (T_2 / T_1) - R \ln (T_2 / T_1) = C_v \ln (T_2 / T_1)$$

$\ln (T_2 / T_1) = (s_2 - s_1)/ C_v = 0.343/0.312 = 1.0986$

$Pv = RT$ => $(P_2 / P_1) (v_2 / v_1) = T_2 / T_1 = P_2 / P_1$

$T_2 = 2.7 \times T_1 = 818.3$, $P_2 = 2.7 \times P_1 = 113.57$



${}_1Q_2 = 1 \times 0.3122 (818.3 - 303.15) = 160.8$ kJ

$m(s_2 - s_1) = \int {}_1Q_2/T_{res} + {}_1S_{2\text{ gen tot}}$

${}_1S_{2\text{ gen tot}} = 1 \times 0.31 - 160.8 / (1300 + 273) = 0.208$ kJ/K

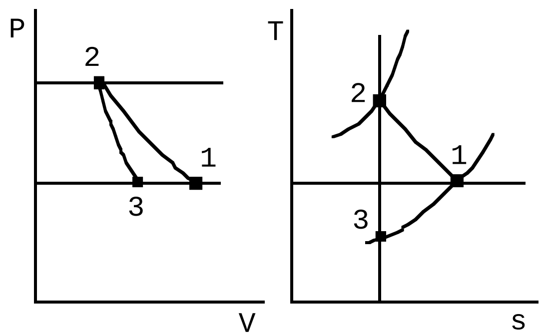
8.66 A cylinder/piston contains air at ambient conditions, 100 kPa and 20°C with a volume of 0.3 m3. The air is compressed to 800 kPa in a reversible polytropic process with exponent, *n* = 1.2, after which it is expanded back to 100 kPa in a reversible adiabatic process.

a. Show the two processes in *P–v* and *T–s* diagrams.

b. Determine the final temperature and the net work.

c. What is the potential refrigeration capacity (in kilojoules) of the air at the final state?

a)



$$m = P_1V_1/RT_1 = \frac{100 \times 0.3}{0.287 \times 293.2} = 0.3565 \text{ kg}$$

b) $$T_2 = T_1(P_2/P_1)^{\frac{n-1}{n}} = 293.2\left(\frac{800}{100}\right)^{0.167} = 414.9 \text{ K}$$

$$_1w_2 = \int_1^2 Pdv = \frac{P_2v_2\text{-}P_1v_1}{1\text{-}n} = \frac{R(T_2\text{-}T_1)}{1\text{-}n} = \frac{0.287(414.9\text{-}293.2)}{1\text{-}1.20} = \text{-}174.6 \text{ kJ/kg}$$

$$T_3 = T_2(P_3/P_2)^{\frac{k-1}{k}} = 414.9\left(\frac{100}{800}\right)^{0.286} = \mathbf{228.9\ K}$$

$$_2w_3 = C_{V0}(T_2\text{-}T_3) = 0.717(414.9 - 228.9) = +133.3 \text{ kJ/kg}$$

$$w_{NET} = 0.3565(-174.6 + 133.3) = \mathbf{-14.7\ kJ}$$

c) Refrigeration: warm to T_0 at const P

$$_3Q_1 = mC_{P0}(T_1 - T_3) = 0.3565 \times 1.004\ (293.2 - 228.9) = \mathbf{23.0\ kJ}$$